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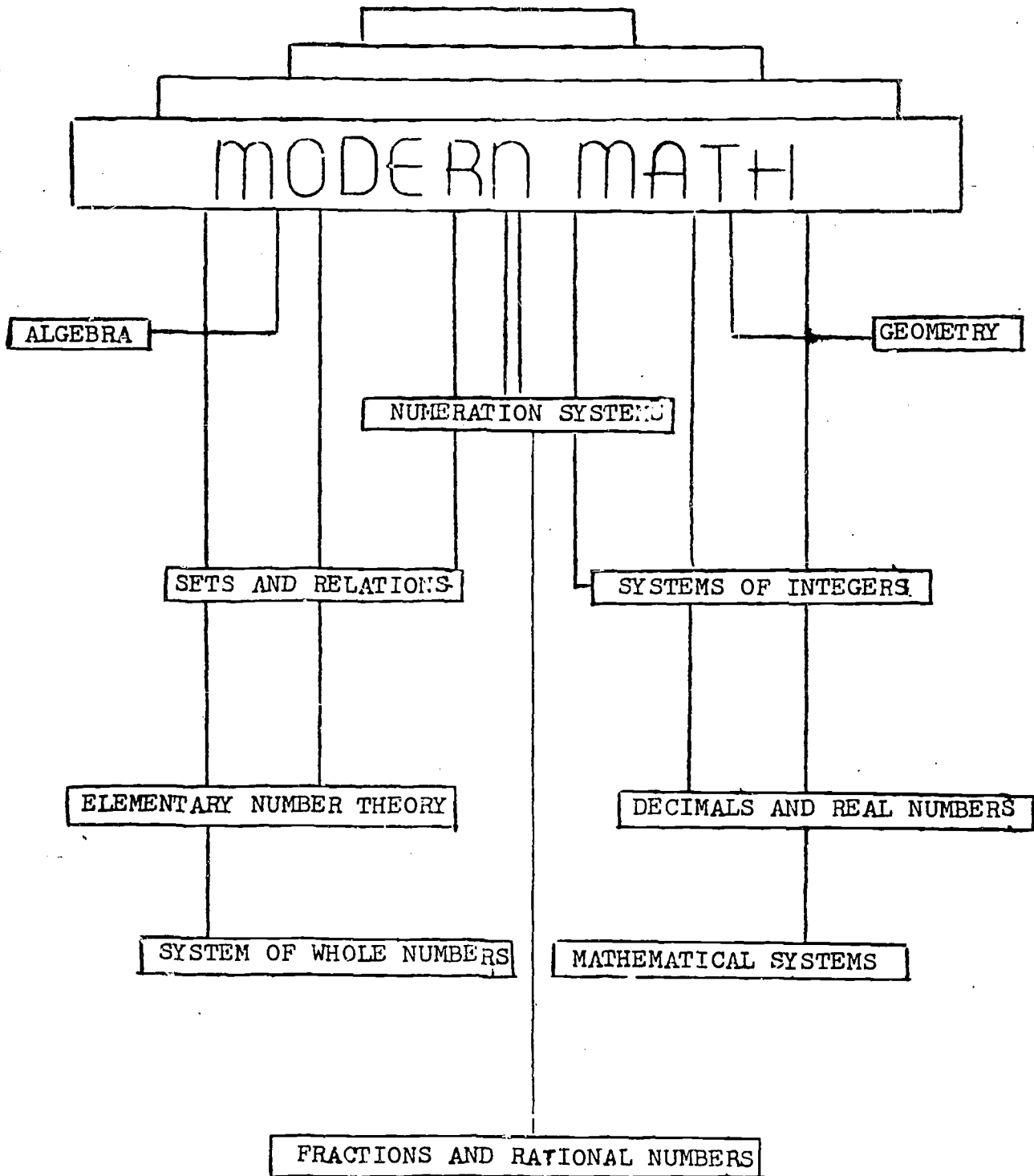
ABSTRACT

This guide includes a list of general objectives and a scope chart of units to be covered in grades K-12. Objectives for specific topics are listed and are coded to the scope chart; text sources and materials are suggested; and lists of learning activities and audiovisual aids are provided. Separate sections of the guide cover topics in elementary school mathematics and junior high school mathematics; an individual lesson program for Algebra II is detailed; and topics to be covered in grade 12 are specified. A list of supplementary mathematics aids and sources is included. This work was prepared under an ESEA Title III contract. (DT)

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SHERIDAN PUBLIC SCHOOLS  
DISTRICT NO. 7  
SHERIDAN, WYOMING

ED 081590



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## A MODEL FOR MATHEMATICS CURRICULUM DEVELOPMENT

A Title III, ESEA Project Developed by: The Sheridan School District #7, Sheridan, Wyoming.

This publication is the initial results of a Title III, ESEA project in Sheridan, Wyoming.

The project was proposed for the purpose of developing a coordinated curriculum in mathematics and social studies through the use of behavioral objectives.

The dissemination of this publication by the State Department of Education is intended to provide the school districts of Wyoming with a model, from which each individual group can develop their own mathematics curriculum.

This guide was developed around the available materials, projected program plans and local philosophy of the Sheridan School District. It is immediately apparent that any curriculum group wishing to use this model must adapt it to their local situation.

Please note that this curriculum publication is offered, not as a finished product, but instead as an on-going handbook which may be revised, re-worked and improved as the needs arise. Curriculum development is by necessity an on-going flexible process, and it is hoped that you, the users of this publication maintain this flexible spirit in your endeavors.

Certain of the activities listed in the section following the behavioral objectives were reproduced from various commercial sources. In some cases these sources are not noted. It is hoped that when these activities are used in classroom quantities the commercial source will be referred to by the teacher before using.

# MATHEMATICS FOR SHERIDAN SCHOOLS

Grades K-12

Curriculum Guide

1969 Edition

Sheridan Public Schools

Sheridan, Wyoming

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## POLICY

We recognize the right of the individual student to develop to the fullest extent of his capacities. It shall be the policy of the mathematics program of the Sheridan Schools that each student experience success in his work and realize a need for understanding and applying mathematics effectively in daily living.

## OBJECTIVES

In accordance with this policy the mathematics program proposes the following overall goals;

1. To enable the student to compute accurately in the fundamental operations of addition, subtraction, multiplication, and division of rational and irrational numbers. The student should have a working knowledge of the properties of numbers.
2. To develop the student's ability to solve problems: (1)interpret facts in any given problems, (2)discriminate between pertinent and nonpertinent facts, (3)visualize the relationship of those facts and form a mathematical sentence using them, (4)determine a reasonable answer by estimation, (5)compute accurately and (6)apply a reliable check.
3. To understand and use numeration systems with particular emphasis on the decimal system.
4. To develop a sequence of logical thinking necessary to understand the structure of mathematics.
5. To identify the relationship of mathematical sentences to applications and problem solving.
6. To analyze and apply mathematical interpretation to graphic representations.
7. To recognize, represent, and interpret geometric shapes.

8. To develop awareness of the practical applications of mathematics and arithmetic computation in everyday living.
9. To acquire basic concepts, commensurate with his ability, which would enable the student to progress to the next level.
10. To develop student's mathematical reasoning ability. This involves both logical and critical thinking, which should lead to wise decision making and open the door to intuitive discoveries in new fields.
11. To encourage flexibility in application of mathematical skills so they can be adapted to the ever-changing needs of our society.
12. To explore the use of statistics, probability studies, and graphical analysis in solving problems in our complex society.
13. To make the student become aware of the interdisciplinary aspect of mathematics, i.e., the correlation of mathematics with other disciplines and the reverse.

## INTRODUCTION

This mathematics guide was produced during a six week summer workshop as a curriculum project of School District #7 with major funding under Title III of the Elementary-Secondary Education Act. It is offered, not as a finished product, but instead as an on-going handbook which may be revised and improved as the need arises. The objective of the guide is to assist teachers by suggesting a variety of materials and approaches to mathematics, which it is hoped will enable every student to experience success in this area.

So the guide may function more successfully, the following points should be noted:

- (1) Only if every teacher uses the guide can a fair evaluation be made of its effect on the mathematics program. Such evaluation will be attempted by comparing the results of the Stanford Achievement Tests given in the fall of 1969 and the fall of 1970, and by informal attitude tests.
- (2) The guide is intended to be a useful teacher handbook. Therefore, additions or revisions are requested. Extra space has been left on each page for this purpose.
- (3) The guide is coded to the scope chart. The right hand side of each page bears the code. Immediately below, the grade levels concerned are indicated. The rectangular box contains the curricular objective and its sequences. The remainder of the page contains the behavioral objective and related activities. The typical breakdown which follows may be helpful in clarification. Coding for the grade levels is as follows:

G - grade. Hence, G4 means grade 4.

I - intuitive

S - standard (mastery)

R - reinforce

E - extend

O - optional

Curricular Objective Sequence	<div style="border: 1px solid black; padding: 5px; display: inline-block;">           Linear and Metric Perimeter         </div>	IV Measures ← Scope  G4 S G5 R ← Grade Level G6 R
Instructional (behavioral) Objective   Readiness pertinent to the objective	Activities 1a.  b.  c.  d.Evaluation	

(4) Behavioral objectives are so stated that the evaluation of the student's achievement is explicit in the objective. However, in many cases, a sample evaluation is offered as an additional help. Since behavioral objectives must, of necessity, state a definite standard of achievement, the objectives in this guide are so written. However, we recognize that each student is an individual and varying degrees of performance may be expected. Therefore, the level of efficiency for each student must be left to the discretion of the teacher.

(5) Each behavioral objective is accompanied by a list of activities and media. Some of these are listed or explained on the same sheet as the objective. In other instances, a reference may be made to a specific activity in the activity section. In every case, the activities so mentioned are to be found immediately following the section or strand and are printed on green paper.

(6) Not all of the materials suggested in the guide are at the present time available to teachers. However, each teacher is encouraged to order recommended materials through his own school as soon as possible.

(7) Many of the activities listed entail the use of media which can be found or made easily. Teachers may wish to assemble these into packets which the individual student may use as part of his learning experience.

## SCOPE

### I. Sets

### II. Numeration and Number Theory

- A. Recognition of numbers
- B. Numeration systems
- C. Place value
  - 1. Decimal system
  - 2. Other number bases
  - 3. Exponents
  - 4. Scientific notation
  - 5. Clock arithmetic (modular)
- D. Absolute Value
- E. Factors
- F. Properties

### III. Real Number Operations

- A. Addition-subtraction
  - 1. Whole numbers
  - 2. Rational numbers
    - a. Common fractions
    - b. Decimals
  - 3. Irrational numbers
- B. Multiplication-division
  - 1. Whole numbers
  - 2. Rational numbers
    - a. Common fractions
    - b. Decimals
    - c. Per cents
  - 3. Irrational numbers
- C. Word-problem solving
  - 1. Symbols
  - 2. Estimation
  - 3. Practical applications (including interdisciplinary)

### IV. Measurement

- A. Linear
- B. Weight
- C. Volume
- D. Time
- E. Temperature
- F. Area
- G. Metric

### V. Money

### VI. Geometry

- A. Plane
  - 1. Recognition of shapes
  - 2. Construction and mensuration
  - 3. Proofs
- B. Space

### VII. Mathematical sentences

- A. Number sentences
- B. Sentences with variables
- C. Equations and inequalities

### VIII. Graphs

- A. Pictorial graphs
- B. Coordinate graphing

### IX. Probabilities and statistics

- A. Descriptive statistics
  - 1. Averages - mean, median, mode
  - 2. Frequency distribution, cumulative, percentile
  - 3. Variance and standard deviation

## **IX. Continued**

- B. Combinations and permutations
- C. Probability

## **X. Trigonometry**

- A. Right triangle trigonometry
- B. Circular functions, defined, wrapping (all six functions)
- C. Reference angles and use
- D. Graph of the trig functions
- E. Identities
- F. Trig equations
- G. Functions of sum, difference, double, and half angles (formulas)
- H. Inverse of trig functions and relations
- I. Polar coordinates and vectors
- J. Sine law and cosine law
- K. Using logs to solve right and oblique triangles.

## **XI. Analytical geometry (see Functions)**

- A. Coordinates
- B. The line--the plane
- C. Functions, relations and graphs
  - 1. Range, domain, function notation
  - 2. Linear function and direct variation
  - 3. Quadratic functions
    - a. Circle
    - b. Ellipse
    - c. Parabola
    - d. Hyperbola - inverse variation
  - 4. Parametric and polar equations
- D. Transformations
  - 1. Rotation
  - 2. Translation

## **XII. Reasoning or Logic**

- A. Logical statements
- B. Variables and quantifiers
- C. Conditional statements
- D. Compound statements and truth tables
- E. Deductive, inductive, and indirect proof
- F. Mathematical induction
  - 1. Sequence and series
    - a. Arithmetic and geometric progressions
    - b. Infinite sequence
  - 2. Binomial Theorem
- G. Digital computer methods.

## **XIII. Functions**

- A. Polynomial functions
- B. Exponential functions and numerical concepts
- C. The logarithmic function
- D. Quadratic Formula
- E. Evaluating the function
  - 1. Nature of roots - complex numbers
  - 2. Synthetic substitution
  - 3. Remainder and factor theorem
- F. Property of continuity - irrational roots
- G. Limit concepts

## **XIV. Matrices and determinants**

- A. Definition
- B. Cramer's Rule

**SHERIDAN PUBLIC SCHOOLS**

**SUGGESTED CHECK SHEET FOR ELEMENTARY MATH**

**Key to Marking:** Circled number means the topic has been introduced  
 Slash within circle means topic introduced and  
 somewhat developed but needs more work  
 X within circle means item is developed to the  
 extent indicated by the behavioral objective

(22)

(22)

(X2)

Objective Number

I.	Sets	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16
II.	Numeration and Number Theory	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
III.	Number Operations Addition-Subtraction Whole Numbers	1 2 3 4 5 6 7 8 9 10 11 12 13 14
III.	Number Operations Multiplication- Division--Whole Numbers	1 2 3 4 5 6 7 8 9 10 11 12
III.	Number Operations Common Fractions	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22
III.	Number Operations Decimals	1 2 3 4 5 6 7 8 9
III.	Number Operations Per Cent	1 2 3 4 5 6 7 8
IV.	Measurement	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
V.	Money	1 2 3
VI.	Geometry	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
VIII.	Graphs	1 2 3 4 5 6 7 8 9 10 11 12

	Teacher's Initial	Year
K.	_____	19__-__
1.	_____	19__-__
2.	_____	19__-__
3.	_____	19__-__

	Teacher's Initial	Year
4.	_____	19__-__
5.	_____	19__-__
6.	_____	19__-__

**PATTERNS**

I. SETS

K-S

1. The learner will show a recognition of patterns by continuing the pattern either by drawing or with concrete objects with 85% accuracy as observed by the teacher.

Readiness: Recognition of like objects.

- 1a. Addison-Wesley-Kdgn-Unit 2
- b. S.R.A.-Kdgn-pp. 13-18
- c. Floor tiles
- d. The children
- e. Felt geometrical shapes
- f. Children's clothing
- g. Beads
- h. Peg board
- i. Blocks
- j. Chalkboard

**POSITIONAL AND COMPARISON RELATIONS**

I. SETS

K-S

2. The learner will compare two objects with respect to size, height, length and shape with 85% accuracy as observed by the teacher.

- 2a. Addison-Wesley-Kdgn-pp. 1-6
- b. S.R.A.-Kdgn-pp. K11-K14
- c. Geometric shaped blocks
- d. Refer to activities #1-8, 22
- e. Refer to materials section



POSITIONAL AND COMPARISON RELATIONS

I. SETS

K-S

3. The learner will identify ordinal relationship of two or more objects with 85% accuracy by teacher observation.

Readiness: Patterns

- 3a. Addison-Wesley-Kdgn-pp. 1-6
- b. S.R.A.-Kdgn-pp. K11-K14
- c. S.R.A.-Grade 1-Unit 4
- d. Refer to activities #4, 6, 24
- e. Concrete objects
- f. Refer to materials section

PRIMITIVE NUMBER CONCEPTS

I. SETS

K-S, G1-S, G2-R

4. The learner will discover similarities and differences of objects in given sets and indicate by telling, pointing or marking which is more or less with 85% accuracy by observation of the teacher.

Readiness: Comparison relationships

- 4a. Addison-Wesley-Kdgn-pp. 13-16, 23-26
- b. Addison-Wesley-Gr. 1-pp. 1-14
- c. Addison-Wesley-Gr. 2-pp. 1-2
- d. S.R.A.-Gr. 1-Unit 1
- e. Jenn Publications Catalog-masters and transparencies
- f. Refer to activities #9-10
- g. Refer to materials section

PRIMITIVE NUMBER CONCEPTS

I. SETS

K-S, G1-S, G2-R

5. Given a set of objects, the student will demonstrate one to one matching by drawing a line between corresponding objects with 85% accuracy by teacher observation.

Readiness: Concept of comparison relationships

- 5a. Addison-Wesley-Kdgn-pp. 17-18  
b. S.R.A.-Kdgn-pp. 19-26  
c. Addison-Wesley-Gr. 1-pp. 3-10  
d. Addison-Wesley-Gr. 2-pp. 1-2  
e. S.R.A.-Gr. 1-Unit 1  
f. Jenn Publications Catalog-masters and transparencies  
g. Refer to activities #11-15  
h. Refer to materials section

6. To show equivalent sets, the student will match by drawing a line between two corresponding objects in given sets with 85% accuracy as observed by the teacher.

Readiness: One to one matching

- 6a. Addison-Wesley-Kdgn-pp. 17-22  
b. S.R.A.-Kdgn-pp. 23-26  
c. Addison-Wesley-Gr. 1-Unit 1  
d. Jenn Publications Catalog-masters and transparencies  
e. Refer to activity #16  
f. Refer to materials section

7. The learner will demonstrate equivalency by drawing a set equivalent to a given set with 85% accuracy as observed by the teacher.

Readiness: One to one matching

- 7a. Addison-Wesley-Gr. 1-p. 20  
b. S.R.A.-Gr. 1-Unit 1  
c. Refer to materials section

UNION OF SETS
---------------

I. SETS

K-S, G1-R,S, G2-6-E

8. The learner will demonstrate union of sets by manipulating two sets of concrete objects into one group with 85% accuracy as observed by the teacher.

Readiness: Understanding of concept of objects in separate sets

- 8a. Addison-Wesley-Kdgn-pp. 39-72  
 b. Addison-Wesley-Gr. 1-pp. 107-108  
 c. Quisenberry rods  
 d. Objects about the room  
 e. S.R.A.-Gr. 1-Unit 1  
 f. Refer to materials section

9. Given two sets of objects, the learner will demonstrate either orally or on a written page the concept of set union with 85% accuracy as observed by the teacher.

Readiness: Manipulating objects into one set and counting

- 9a. Addison-Wesley-Kdgn-Unit 5  
 b. Addison-Wesley-Gr. 1-Unit 6  
 c. Addison-Wesley-Gr. 2-pp. 31-32, 35, 37  
 d. Addison-Wesley-Gr. 3-pp. 48-49  
 e. Addison-Wesley-Gr. 4-pp. 48-49  
 f. Addison-Wesley-Gr. 5-pp. 174-175  
 g. Addison-Wesley-Gr. 6-p. 92  
 h. S.R.A.-Gr. 1-Unit 1  
 i. Venn Diagrams  
 j. Materials section

SUBSETS

I. SETS

K-S, G1-S, G2-R

10. Given a set of concrete objects the learner will demonstrate the concept of subsets by removing like objects to form a subset with 85% accuracy as observed by the teacher.

Readiness: Union of sets

- 10a. Addison-Wesley-Kdgn-pp. 65-72  
b. Addison-Wesley-Gr. 1-Units 7-8  
c. Addison-Wesley-Gr. 2-Unit 3  
d. Flannelboard  
e. Objects in the classroom  
f. S.R.A.-Gr. 1-Unit 1  
g. Refer to Activities #17-18  
h. Refer to materials section

EMPTY SET

I. SETS

K-S, G1-6-R

11. The learner will recognize the empty set as a set containing no objects.

Readiness: Counting and primitive number concepts

- 11a. Addison-Wesley-Kdgn-pp. 35-38  
b. Addison-Wesley-Gr. 1-p. 35  
c. Addison-Wesley-Gr. 3-p. 49  
d. Addison-Wesley-Gr. 4-p. 49  
e. Addison-Wesley-Gr. 5-p. 175  
f. Addison-Wesley-Gr. 6-p. 92  
g. Refer to materials section

INEQUALITIES AND ORDER, AND EQUALITIES

I. SETS

G1-I,S

G2-3-R

G4-6-E

12. The learner will compare two sets of unequal numbers using the symbols for greater than and less than ( $>$ ,  $<$ ) with 85% accuracy.

Readiness: Grouping concrete objects

- 12a. Addison-Wesley-Grade 1-Unit 5  
b. Addison-Wesley-Grade 2-pp. 17-30, 185-188, 195-196, 208-209  
c. Addison-Wesley-Grade 3-pp. 96-97  
d. Refer to activities #19-20, 23  
e. Refer to materials section

13. The learner will compare two sets of identical objects using the symbol for equality ( $=$ ) with 85% accuracy.

Readiness: Inequalities and Inequivalency

- 13a. Addison-Wesley-Grade 1-pp. 113-116  
b. Addison-Wesley-Grade 2-p. 221  
c. Refer to materials section

PRODUCT SETS

I. SETS

G2-I

G3-S

G4-R

14. Given a pictorial set of objects, the learner will demonstrate the concept of product sets by pairing objects and forming as many combinations as possible with 85% accuracy.

Readiness: Addition and counting.

- 14a. Addison-Wesley-Grade 2-pp. 261-264  
b. Addison-Wesley-Grade 3-pp. 124-128  
c. Addison-Wesley-Grade 4-pp. 122-125  
d. Refer to activity #21  
e. Refer to materials section

INTERSECTION OF SETS

I. SETS

G4-S

G5-6-R,E

15. Given two sets of objects, the learner will demonstrate either orally or written the concept of set intersection with 85% accuracy.

Readiness: Set union

- 15a. Addison-Wesley-Grade 4-p. 48  
b. Addison-Wesley-Grade 5-pp. 174-175  
c. Addison-Wesley-Grade 6-p. 92  
d. Venn Diagrams  
e. Refer to materials section

SYMBOLS FOR UNION, INTERSECTION  
AND THE EMPTY SET

I. SETS

G5-6-S

16. The learner will demonstrate a knowledge of the symbols for union, intersection and the empty set by use in problem solving with 100% accuracy.

Readiness: Previous work with sets.

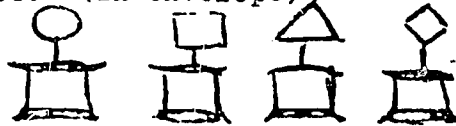
- 16a. Addison-Wesley Grade 5-p. 174  
b. Addison-Wesley Grade 6-p. 92  
c. Refer to materials section

## ACTIVITIES & SETS

### 1. Spool Match - K-1

Materials: 12 spools with a 4" long dowels inserted. Cut shapes out of construction paper such as : star, circle, rectangle, diamond, crescent, oval, etc. (In envelope)

Example:



Directions: This is a matching game that may be played when your work is finished. Place the spools on your desks. Take all the shapes and match them with the spool with the same shape.

### 2. Puzzle Fun - K-1

Let the children make jigsaw puzzles by pasting a colorful picture from a magazine onto cardboard. Take a felt marking pen and scribble a pattern on it to be cut out into pieces. (This is especially good for eye-hand coordination, spatial relationships and shape and size discrimination.)

### 3. Big and Little - K

Each child is given clay. Have the children make objects showing big and little. Let the children decide which is big or which is small.

### 4. Five in a Row - 1-2

Call 10 people to the front of the room. Tell the children to listen carefully and see if they can follow directions. Give directions such as:

1. Will the second person clap his hands.
2. Will the fifth person turn around three times.
3. Will the first person jump up and down.
4. Will the fourth person wave one hand.
5. Will the third person touch his toes.

### 5. Big or Little - 1-2

Children will need paper, pencil and crayons.

Directions: You may play a game called Big or Little. To play this game you must first fold your paper in half the long way (demonstrate). Next fold it in half the short way (demonstrate). You have six squares now. In each square draw a picture of two exact objects making one big and one little. Under each big picture write the word big; under each little picture write the word little. These words are written on the board to help you. If you have enough time color your pictures.



## 6. Positional Skills - K

- (1) Have children work with sets of cutout shapes and arrange them in order of increasing or decreasing size.
- (2) Help children line up according to height--can go from tallest to shortest or shortest to tallest.
- (3) Puzzles which require children to fit graduated circles, squares, etc. in corresponding holes.

## 7. Perception Skills - K-1

Six year olds enjoy matching concrete objects. They are also developing skills and the relationship of objects, as "before, behind, below, above "

At first use the overhead projector and two popsicle sticks. The children at their desks, place their sticks in the same position as those on the screen. When they have mastered placing two sticks in position as those on the screen. When they have mastered placing two sticks in position, use three sticks, and so on. Child can then match sticks to other shapes.

## 8. Positional and Comparison Relations - K-1

Use simple seasonal drawings to develop arithmetic vocabulary concepts. Two jack-o-lanterns in different sizes, for example, can show "big and little, large and small". A cat on a fence and one on the ground can illustrate high and low, while a flying owl over a moon and one sitting in a tree under it describe "over and under."

## 9. Spool to School - K-2

Materials: Have 2 square racks of  $\frac{3}{8}$  inch plywood and 100 poultry netting staples  $\frac{3}{8}$  by  $\frac{3}{8}$  inches as follows: Place staples  $1\frac{1}{2}$  inches apart, which allows  $1\frac{1}{2}$  inch border. With pliers squeeze staples almost together at the sharp end before pounding them into the board in ten rows of ten.

Directions: Divide the class into two "collecting teams," as the spools come in each day the children place them on the rack belonging to their team, putting them in rows of ten. Discuss which team has more or less until one team wins.

These spools can be later sprayed with enamel and could be used for many number games.

## 10. More or Less - 1-2

Players: Groups of two

Materials: Sets of number cards 1-10 each showing one number or symbols or pictures.

Directions: At a signal each child turns up a card. The child whose card has the larger number is asked to tell how much larger his number is than the other. If he is correct, he gets a point. The child who first gets 10 points is the winner. (Good for a rainy day

# 11. One to One Correspondence - K-1

Use paper dolls or magazine figures to give practice in number matching. On the front of a pair of figures (two girls, boy and girl, mother and father, grandma and grandpa, and so on,) write the same number. Each pair has a different number. Mix up the dolls and put all together in a box. In free time a child can pair up the dolls, making sure that the numbers match.

# 12. Bounce the Ball - K-2

Materials: Tennis or rubber ball

Directions: The teacher says the number, such as 15. She then bounces the ball while the children count to themselves. Before the designated number is reached, she calls upon a child to complete the bounces. This child in turn starts a new series and calls on another child to complete these bounces.

# 13. Number Party - K-1

Materials; Played in groups of six to twelve. Name cards for each child, two each. Each child holds one of his own cards. The matching cards are held by the "party giver" to be used in checking the number of letters in each name.

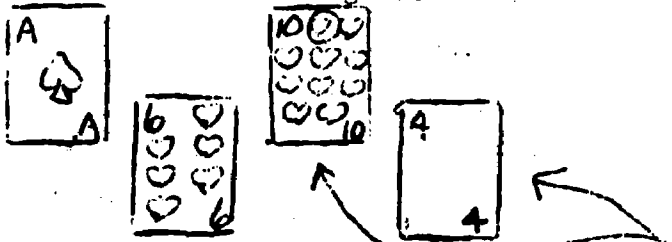
Directions: The "party" is composed of several class members, of whom one is appointed the "party giver." One child comes to the party and taps five times on the wall or door because there are five letters in her name (Sarah). If she taps correctly the party giver says, "Come in Sarah." If not she taps again. A new "party giver" and "party comer" are appointed and game proceeds as before.

# 14. Fingers are Handy - 1-2

Ask a child to name the members of his family, putting up one finger for each. Another child makes a tally mark on the chalkboard for each finger raised. The first child then counts the tally marks and writes the correct number. Continue with each child.

# 15. One to One Correspondence - 2-3

Ten cards are laid out in a pattern corresponding to the symbols on the cards. A dealer thinks of a card on the board and points to its position on the ten card. The other children guess which card on the board he is thinking of.



Ex: Dealer points to this position on 10 card, therefore the card he is thinking of is the 4 of spades.

16. Pupils are given (or they may draw) pictures of some object: animal, machine, vehicle, person, etc. The leader then names an activity such as: "They can run"; "You can ride them"; "You put animals in them"; or "They haul things for us." This may be played as a quick response game, something like "Simon Says." The set categories may be prepared in advance for young children, and the leader may draw one from a deck of category cards placed face down on a table. Pupils holding appropriate cards stand.

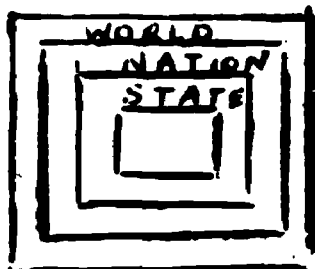
Teachers may use the game as a competitive exercise, or they may simply observe the reactions of pupils as different sets are called.

#### 17. Fox and Grapes - 2-3

Materials: nine corks

Directions: The teacher tells story: "One day a fox found nine grapes. He tried to think of all the ways he could group his grapes before eating them. Let's help the fox." One child may say "He could eat three today, three tomorrow, and three the next day" (Show groups with corks.) Get as many ideas as possible from the group.

#### 18. Subset Chart



This shows that "Billy" is a member of the set "school" which is a subset of the set "town" and so forth.

#### 19. Greater Than--Less Than - k-2

To review the concept of "greater than" and "less than", have several children who do not really understand the idea step one step forward to show "greater than" and one back to show "less." Use floor tiles number 1-9.

#### 20. K-2

Have a child use a large number line on the floor. The child places both feet on the number four. He steps one step forward and then says, "Five is greater than four." Then have the child step back one step and say, "Three is less than four."

#### 21. Product Sets. - 2-4

The teacher asks how many different three-figure numbers can be written with three different figures. Example: "How many three figure numbers can be written with the figures 1, 2, 3? The children should make a guess before writing. They should also remember that each number should have three figures and a figure should only be used once in a number. Six combinations are possible: 123, 321, 213, 231, 312, 132.

## 22. Shape Hunt - K-1

**Materials:** Cut 6 different geometric shapes from construction paper. Cut two of each shape. While children are out for recess, hide one set around the room.

**Directions:** I hid 6 shapes somewhere in the classroom. They look like this (hold up the matching set of six). I am going to give 6 people one of these shapes. Each person must look until the matching shape is found. If you see one that is not exactly like yours, walk right by and say nothing. Let's see how long it takes to find all 6 shapes. After all are found, children put their heads on their desks and the shapes are hidden again.

## 23. Which are the Same? - 2-3

**Materials:** Printed worksheets. Write on the printed pages pairs of numbers, some of which match exactly and some that differ slightly.

**Directions:** Circle the number that is exactly alike.

**Example:**

1.	43	43
2.	39	93
3.	24	24
4.	533	536
5.	502	502

## 24. Catcher's Mitt - K-1

**Materials:** 20" squared tagboard--2" squares, numbered 1-100. 1 catcher's mitt drawn on tagboard with a hole in the middle.

**Example:**



**Directions:** We will play catch with numbers. One person will come to the front of the room. I will tell you what you and how many over. When I "throw" you a number you must catch it with your mitt by moving it to the correct place, putting the hole over the number. If you are correct, you will get a point. When you get three points you may choose another player. This could be played in teams.

SETS.

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RECOGNITION OF NUMBERS

II. NUMERATION AND  
NUMBER THEORY

K-S

G1-R, S

G2-R

1. The student will demonstrate the sequential order of 0-9 cardinal numbers, either orally, written, or with concrete objects with 100% accuracy.

Readiness: Recognition and construction of numerals

- 1a. Addison-Wesley-Kdgn-Unit 4
- b. Addison-Wesley-Grade 1-pp. 45, 51-52
- c. Addison-Wesley-Grade 2-Unit 1
- d. Refer to activities #1-24
- e. Concrete objects--blocks, chairs, beads, etc.
- f. Cuisenaire rods
- g. No. 525 Number Concept Chart from Ideals
- h. No. 620 Number Readiness Posters from Ideal
- i. Refer to materials section

2. Given a set of concrete objects or pictures, the student can identify, name orally, and write the numeral for the cardinal numbers of the set with 100% accuracy as observed by teacher.

Readiness: Understanding and recognition of sets.

- 2a. Addison-Wesley-Kdgn-Units 4 & 5
- b. Addison-Wesley-Grade 1-Unit 2
- c. Addison-Wesley-Grade 2-p. 3-6
- d. S.R.A. Kdgn-Unit 3
- e. Refer to activities #8, 10, 11, 14, 16, 17, 20, 23, 25
- f. Concrete objects
- g. Abacus
- h. Cuisenaire rods



RECOGNITION OF NUMBERS

K-S, G1-S, G2-R

3. Given a verbal or written numeral such as 7, the learner can construct and identify sets containing 7 members with 100% accuracy.

Readiness: Identification of numerals

4. The learner will demonstrate the concept of odd and even numbers by counting orally or by completing a sequence of odd and even numbers with 85% accuracy.

Readiness: Oral and written number concepts.

- 3a. Addison-Wesley-Kdgn-Units 4 & 5  
b. Addison-Wesley-Grade 1-Unit 3  
c. Addison-Wesley-Grade 2-p. 5  
d. Refer to activities #1-24  
e. Cuisenaire Rods

- 4a. Addison-Wesley-Grade 1-pp. 269-270  
b. Addison-Wesley-Grade 2-pp. 241-244  
c. Addison-Wesley-Grade 3-pp. 290-293  
d. Addison-Wesley-Grade 4-p. 220  
e. Addison-Wesley-Grade 5-p. 55  
f. Refer to activities #11, 25-29, 36, 38  
g. Concrete objects

RECOGNITION OF NUMBERS

G1-S, G2-3-R

5. The learner will demonstrate skip counting of 2's, 3's, 4's, 5's, 10's, 50's and 100's by counting orally and by completing a series of given numbers with 85% accuracy.

Readiness: Odd-even numbers

- 5a. Addison-Wesley-Grade 1-pp. 271-274  
b. Addison-Wesley-Grade 2-pp. 199, 245-246  
c. Addison-Wesley-Grade 3-pp. 33, 39, 123, 137, 140  
d. Refer to activities #26, 28, 29, 36, 38

ROMAN NUMERALS

G2-S-  
G3-6-R

6. Given the numerals 1-12, the student can read and write the Roman numerals I-XII.

Readiness: Understanding of Arabic numerals.

- 6a. Addison-Wesley-Grade 2-pp. 137-138  
b. Refer to activities #30-33

7. The learner will read, write and know the meaning of the Roman numerals L, C, D, and M.

Readiness: Previous work with Roman numerals.

- 7a. Addison-Wesley-Grade 5-pp. 14-15  
b. Refer to activities #30-33

II. NUMERATION AND  
NUMBER THEORY

OTHER SYSTEMS

G5-6-Op.

8. Given a series of worksheets, the learner will recognize and write the correct number, by using the Egyptian-Greek and East Arabic numeral tables.

Readiness: Roman numerals.

- 8a. Addison-Wesley-Grade 5-p. 15  
b. Addison-Wesley-Grade 6-p.1  
c. "The Story of Figures," published by Burroughs Corporation

II. NUMERATION AND  
NUMBER THEORY

PLACE VALUE

G1-S  
G2-R

9. Given a set of 10 concrete objects, the learner will recognize the concept of ten by counting orally with 85% accuracy.

Readiness: The cardinal number 0-9

- 9a. Addison-Wesley-Grade 1-pp. 153-156  
b. Addison-Wesley-Grade 2-pp. 7-8  
c. Refer to activities #1-24  
d. Concrete objects  
e. Abacus  
f. Cuisenaire rods

II. NUMERATION AND  
NUMBER THEORY

PLACE VALUE

G1-S

G2-R

10. Given sets of concrete objects containing 30, 40, or 50 objects, the learner will demonstrate the concept of grouping by 10's by counting sets of ten with 85% accuracy.

Readiness: Concept of ten.

- 10a. Addison-Wesley-Grade 1-Unit 7  
b. Addison-Wesley-Grade 2-pp. 7-9  
c. Refer to activities #35-39  
d. Concrete objects  
e. No. 7305 Counting Frame from Ideal

11. Given a set of concrete objects (such as 43) the child will group the objects by 10's showing the concept of grouping by tens when the number in the set is not a multiple of ten with 85% accuracy.

Readiness: Grouping multiples of ten.

- 11a. Addison-Wesley-Grade 1-Unit 7  
b. Addison-Wesley-Grade 2-Unit 2  
c. Refer to activity #35  
d. Concrete objects  
e. Two Place Number Board - \$3.30 from Ideal - No. 758

II. NUMERATION AND  
NUMBER THEORY

PLACE VALUE

G1-S

G2-R

12. Given a worksheet of pictorial sets, the student will write the corresponding two-digit numeral with 85% accuracy.

Readiness: Grouping by ten.

- 12a. Addison-Wesley-Grade 1-Unit 7  
b. Addison-Wesley-Grade 2-Unit 2  
c. Refer to activity #35

13. The learner will demonstrate the order concept within a given decade by counting and writing 0-99 with 85% accuracy.

Readiness: Grouping by ten.

- 13a. Addison-Wesley-Grade 1-pp. 59-88  
b. Addison-Wesley-Grade 2-pp. 9-22  
c. Refer to activities #1-24, 36-39  
d. No. 522 One Hundred Chart from Ideal  
e. No. 757 Counting Bar K-1

PLACE VALUE

G1-I, G2-S, G3-R

14. Given a two-digit numeral (such as 67) the student can identify name, write and distinguish the numerals that are in the ones and tens places with 85% accuracy.

Readiness: Grouping by ten.

- 14a. Addison-Wesley-Grade 1-pp. 255-258  
b. Addison-Wesley-Grade 2-pp. 7-30, 155  
c. Addison-Wesley-Grade 3-pp. 2-8  
d. Refer to activity #35

15. Given a two-digit numeral (such as 82) the student will write the expanded numeral ( $80 + 2$ ) with 85% accuracy.

Readiness: Grouping by 10.

- 15a. Addison-Wesley-Grade 1-Unit 7  
b. Addison-Wesley-Grade 2-Unit 2  
c. Addison-Wesley-Grade 3-pp. 26-28  
d. Addison-Wesley-Grade 4-pp. 26-27

II. NUMERATION AND  
NUMBER THEORY

PLACE VALUE

G2-S  
G3-4-R

16. Given a three-digit numeral (such as 317), the learner can identify, name, write and distinguish the numerals that are in the one's, ten's, and 100's places with 85% accuracy.

Readiness: Two-digit numbers.

- 16a. Addison-Wesley-Grade 2-pp. 27-30, 179-187  
b. Addison-Wesley-Grade 3-pp. 6-8  
c. Addison-Wesley-Grade 4-pp. 28-29  
d. Refer to activities #36-40  
e. No. 755 Place Value Chart from Ideal  
f. No. 748 Modern Computing Abacus from Ideal  
g. No. 747 Number Grouping Frame-K-3

17. Given a three-digit numeral (such as 743), the student will write the expanded numeral ( $700 + 40 + 3$ ) with 85% accuracy.

Readiness: Two-digit expanded notation.

- 17a. Addison-Wesley-Grade 2-pp. 179-187  
b. Addison-Wesley-Grade 3-pp. 9-11  
c. Addison-Wesley-Grade 4-pp. 28-29



PLACE VALUE

G2-I, G3--4-S

18. Given a four, five or six digit numeral, the student can identify, name, write, and distinguish the numerals that are in the 1's; 10's; 100's; 1000's; 10,000's; 100,000's places with 85% accuracy.

Readiness: Three-digit numerals.

19. Given a seven, eight, or nine digit numeral, the student will identify, name, write, and distinguish the numerals that are in the 1's; 10's; 100's; 1000's; 10,000's; 100,000's; 1,000,000's; 10,000,000's; or 100,000,000's places.

Readiness: All previous place value work.

- 18a. Addison-Wesley-Grade 2-pp. 188-199  
b. Addison-Wesley-Grade 3-pp. 11-18  
c. Addison-Wesley-Grade 4-pp. 30-31, 36-37

- 19a. Addison-Wesley-Grade 4-pp. 38-41  
b. Addison-Wesley-Grade 5-pp. 4-6

II. NUMERATION AND  
NUMBER THEORY

PLACE VALUE

G4-I,S

G5-Jr. HI.-S,R

20. Given any numeral through trillions, the student will identify, name, write and distinguish the numerals that are in the ones through the trillions places.

Readiness: All previous place value work.

- 20a. Addison-Wesley-Grade 4-pp. 44-45  
b. Addison-Wesley-Grade 5-pp. 7-9  
c. Addison-Wesley-Grade 6-pp. 1-7  
d. Addison-Wesley-Jr. High  
First course-pp. 1-17  
Second course-pp. 1-20  
Third course-pp. 1-13

II. NUMERATION AND  
NUMBER THEORY

EXPONENTS

G6-S

21. The learner will recognize exponents and re-name them using exponential notation (such as:  $10 \times 10 = 10^2$ ).

Readiness: Work in place value and expanded notation.

- 21a. Addison-Wesley-Grade 6-pp. 10-13  
b. Refer to activity #41

II. NUMERATION AND  
NUMBER THEORY

OTHER BASES

G5-S, G6-S

- |  |  |
|--|--|
| <p>22. Given a numeral of another base (14<sub>5</sub>), the learner will read "one, four base five" <u>avoiding</u> the base ten terminology--"Four-teen" with 100% accuracy.</p> | <p>22a. Addison-Wesley-Grade 5-pp.10-13<br/>b. Addison-Wesley-Grade 5-pp.14-18<br/>c. Houghton-Mifflin-Grade 6-pp.24-25, 184-187, 266, 329<br/>d. Refer to activities #42-44</p> |
| <p>23. The learner will count orally or write the sequence of numbers in a base other than ten with 85% accuracy.</p> <p>Readiness: Base ten.</p>                                  | <p>23a. Addison-Wesley-Grade 5-pp. 10-13<br/>b. Addison-Wesley-Grade 6-pp. 14-18<br/>c. Refer to activities #42-44<br/>d. The Dienes Multi-base Arithmetic Blocks #28008</p>     |

II. NUMERATION AND  
NUMBER THEORY

OTHER BASES

G5-I

G6-Jr. Hi.-S

24. Given a set with 1-10 members, the student can group the members and write the numeral for other bases (such as five) for the cardinal number of the set. This should be used to strengthen base ten.

Readiness: Base ten.

- 24a. Addison-Wesley-Grade 5-pp. 10-13  
b. Addison-Wesley-Grade 6-pp. 14-18  
c. Addison-Wesley-Jr. Hi.  
Second course-pp. 21-25  
Third course-pp. 20-25  
d. Refer to activities #42-44  
e. The Dienes Multi-base Arithmetic  
Blocks #28008

II. NUMERATION AND  
NUMBER THEORY

PRIME AND COMPOSITE NUMBERS

G3-I,S

G4-6-S

25. Given the numerals 1-100 the learner will identify which numerals are prime numbers and which numerals are composite numbers with 85% accuracy.

Readiness: Multiplication and division skills.

- 25a. Addison-Wesley-Grade 3-pp. 296-297  
b. Addison-Wesley-Grade 4-pp. 226-227  
c. Addison-Wesley-Grade 5-pp. 172-173  
d. Addison-Wesley-Grade 6-p. 90  
e. Refer to activity #45  
f. Filmstrip "Factors and Primes"  
No. 101-2, from Ideals

ACTIVITIES - NUMERATION  
& NUMBER THEORY

1. Hull Gull - K-1

"IT" takes a number 1-10 of beans from a box of beans while "player" looks away and closes his eyes. "IT" extends his closed hand and says "Hull-Gull handful. How many?" "Player" guesses a number from 1-4, "IT" opens his hand and they count the number together. If "player" guesses correctly, then the two change places. If not "IT" remains "IT" and the game continues.

2. Concept of Numbers 1-5 - K

On about twenty strips of cardboard write the numbers from 1-5. These are placed upside down in two plastic cups. Children form two lines. The first one in each line draws a card, reads the number and takes that many steps forward (could hop, jump and so on). The line getting across the room or playground first wins.

3. Number Party - K-1

Use this game to give practice in counting. One child sits in a corner. Another comes to him and taps on the floor or wall. The one in the corner says, "Come in, four (or whatever the number is)." He sits beside the first child. Another comes and so on. Children must tap distinctly. When the one in the corner counts wrong, he must change places with the person tapping.

4. Pony Trot - K-1

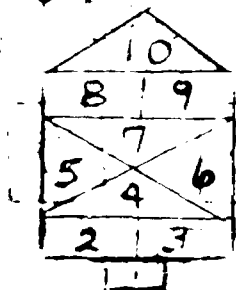
Materials: Number cards (4" x 4") from 1-10 (do in groups)

Directions: The children stand in a circle, each child holding a number card. The "Ringmaster" calls a number. The "ponies" with that number, step to the center and tap their feet to correspond with their number. If right the "Ringmaster" says "Trot ponies." The "ponies" trot around the circle once and back to their places. Continue until all ponies have trotted.

5 Hop Scotch - K-1

Materials: Large pattern drawn on floor, sidewalk, or a large sheet of paper.

Example:



Directions: The child jumps from section to section calling out the numerals as they jump. The child who reaches 10 without touching a line or touching his other foot to the ground wins. Could be played in reverse from 10 to 1.

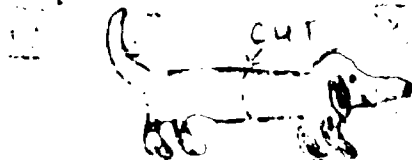
6. I Spy - K-2

Materials: Pocket chart and numerals from 1-10 written in manuscript on pieces of Oaktag - 4" x 4".

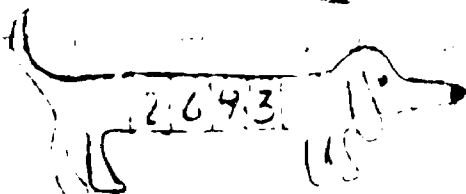
Directions: Place cards in the pocket chart in a scrambled fashion. The teacher says, "Find the number that comes between 4 and 6." The child picks up 5, says "Five," and returns it to the pocket chart. Continue this having a child ask the next number.

7. Numbers - K-1

Materials: Cut a dachshund from tagboard. Cut the dog in two as shown by the dotted line in the example. Make 10 flashcards labeled from 1-10.



Directions: Here is a game you may play in free time. You make the dog grow longer. Put the dog on your desk. Pick up one number card. If you can say the number, you may place the card in the dog. (demonstrate) Then pick up the next and so on. See how long you can make the dog grow.



## 8. Sad Bozo - 2-3

**Materials:** 31 cards of tag board--on one card draw a sad clown, on the remaining cards make three matching sets (one showing the numerals from 1-10, another showing the written word from "one to ten," and the last showing objects from 1-10,

Example:



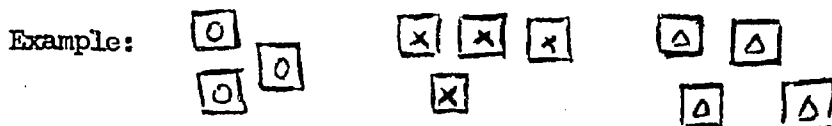
**Directions:** Three or four players sit around a table---one child deals all the cards. Each child holds his cards in his hand. On his turn he draws a card from any player. The object of this game is to collect matched sets of three cards. A set is: a numeral word, a pictured group, the number card; show children a matched set. Each time a child gets a matched set he puts those cards on the table in front of him. When everyone has used his cards, the winner is the one with the most matched sets. The loser will have Sad Bozo.

## 9. Dot to Dot - 1-2




Have the children create their own dot to dot pattern. Most children will make abstract pictures, some will be able to plot actual pictures. This gives the child practice not only in recognizing numerals but in writing them in order too.

## 10. Count the Block - 1-2

**Materials:** Draw on board groups of various marked blocks.



Children need work sheet like example.

- Example:
1. How many ? \_\_\_\_
  2. How many ? \_\_\_\_
  3. How many ? \_\_\_\_

**Directions:** Children must count, then write the numeral.

### 11. Cross the River - K-2

Materials: 1 large sheet of oak tag with irregular riverbank drawn. Ten stones from construction paper with set patterns from 1-10 drawn.

Example:



Directions: Separate class in two teams. Have a member of team 1 try to jump stone to stone in proper sequence. Continue with both teams. Each time a player gets across they score a point. Change stones from time to time. This would be good to use for odd and even numbers and skip counting.

### 12. Concept of Numbers

Materials: Twenty strips of cardboard with the numbers from 1 to 5, two plastic cups.

Directions: Children form two lines; the first one in each line draws a card, reads the number, and takes that many steps forward. (could hop, jump, skip). The line getting across the room or playground first wins.

### 13. Hanging Out the Wash - K-1

Materials: Cut patterns such as pants, shirts, skirts, socks, etc. On these print the numbers from 1-10. Clothesline & clothespins

Directions: Divide into two teams. Pupils from each team take turns selecting a garment from the basket. If the pupil knows the number, the garment is dry, and is put on the table. If not, it must be hung on the line. Team with fewest wet garments win.

### 14. Numeration

Uses for detergent bottles are many, but the unused caps sometimes just collect. These may be used as counting devices in kindergarten. A large paper numeral (from 2 to 10) is placed on a table. Choose a team to arrange colorful caps to make that number. If the numeral is 5, the children might show it with 2 green and 3 white tops, or 4 pink and 1 green.

### 15. Sand Writing - 1-2.

Materials: A table with a cookie sheet or jelly roll pan containing sand. Near the tray, display a chart showing the correct formation of the numbers from 1-10.

Directions: Look at the chart by the sand tray. Take your finger and practice writing one number over and over again in the sand until you think you can write it just right. Then try another number.



16. Grab Bag - K-1

Materials: Twenty-Five 3 x 5 cards with drawn or pasted pictures of 1-5 objects. Place in a box.

Directions: Child picks a card from the box, looks at it, and immediately gives the number associated with the object shown. If correct, he keeps the card. If incorrect, he puts it back in the box. When all the cards are drawn, the child with the most cards wins.

17. Pegs - K-1

Materials: Cards on which numbers are written. Sticks, beans or other markers.

Directions: Each child at his desk is given several cards and a handful of markers. The children place the correct number of markers on each number card. (six markers on the card on which the number 6 appears, etc.)

18. Guessing Game - K-1

Materials: Card holder and cards on which numbers appear.

Directions: A series of consecutive numbers are placed in the card holder such as: 4, 5, 6, 7, 8. One child hides his eyes while another child removes one of the cards. The child who is "IT" tries to name the missing number as he looks at the remaining cards.

19. Fireman - K-1

Draw a large house on the blackboard with smoke coming out of the window to indicate that the house is on fire. Draw a ladder next to the house with each step numbered. To rescue someone in the burning house, a player must read the numbers up the ladder and down again. For each player who rescues a person, a fireman's hat is drawn on the board.

20. Postman - K-1

Make two sets of number cards. Choose one player to be postman and give him one set of number cards. Place the other cards on the "houses" (desks) of the other players. The postman must match his cards with those on the "houses". When he misses, the person who lives in the "house" becomes the new postman.

21. Bounce the Ball - K-2

One child stands in front of the group. He bounces a ball any number of times from one to ten. Other children listen. He calls to another listener to tell how many times the ball was bounced. The child who gives the correct number is permitted to bounce the ball next.

## 22. Number Party - K-1

Several children sit in a corner. Other children have numbers as names. One of the latter comes to the first group and says "May I come to your party?", and taps his number on the floor or wall. The children in the corner say, "Yes, Five, or whatever number, you can come in." Other guests approach in the same way. The children in the corner may take turns inviting the other children to the party.

## 23. Basketball - K-1

Materials: Wastebasket or box, five to ten beanbags.

Directions: Set wastebasket or box in an open space. Draw a line six to eight feet distant. Have players stand behind the line and take turns throwing the beanbags into the basket. Each child gets five to ten throws. He then counts the number of beanbags in the basket to figure his score.

## 24. Buzz - K-2

Children stand in a row. The first child says "one." The second child says "two." The third says "three." The fourth says "four." The fifth child says "BUZZ." The sixth child says "six," and so on through 10. Then start with one again. Buzz may be substituted for any number, for all even or odd numbers, multiples of given numbers (at a higher level) or numbers divisible by certain numbers (also at a higher level).

## 25. Odd or Even - 1-2

Divide the class into two evenly matched teams. One team called "ODD" and the other "EVEN". One member from each team will come to the front of the room and stand back to back. Each child will hold up any number of fingers on one hand. Each player must not peek to see how many his opponent has raised. The rest of you will quickly count how many all together. If the total is an odd number, that team scores a point, etc. The next player comes forward and the game continues. The team with the most points wins.

## 26. Line 'Em Up 1-2

Place flashcards with the numbers 1-20 printed on them along the chalkboard. Make sure that they are not in proper sequence. Have one child come up and put all the "odd" numbers in proper sequence--or all the "even" numbers in proper sequence. When the child finishes he mixes them up and calls on another child.

Game could be varied by having cards from 1 to 100 and have them count by 2's, 5's, or 10's.

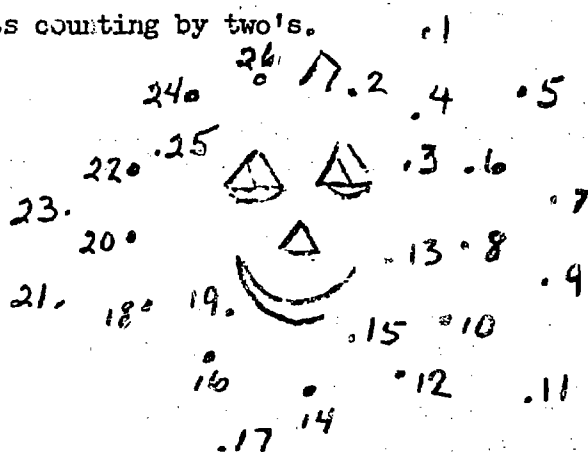
27. Ring Toss 1-2

Materials: A stand-up board with numbered hooks and 3 fruit jar rings.

Directions: Divide into two teams, "odd" and "even". Each child tries to toss his rings at either odd or even numbers; depending on which team he is on. If he gets one on he gets 1 point, 2 = two points, 3 = three points. The team with the most points wins. Team Captain could tally points on the board. Could have 2 boards going at one time.

28. Skip Counting 1-2

Follow the dots counting by two's.



29. Number Down 1-2

Divide the class in 2 groups standing on opposite sides of the room. Give a number to the first person on team A. He must continue counting by 2; 5, 10, or what ever direction given until the teacher says stop. If correct, he remains standing, if not he sits down. Then do the same to group B. The team with the most standing wins.

30. Be A Roman 2-6

Write on the board the Roman numerals from 1-50 (or depending on the grade level). Have one student put his head on the desk, while another child erases 3 of the Roman numerals. The one with his head down will go to the board and write the correct numerals in the places where they have been erased. If he does this correctly, he may be the one to erase the numerals next, while someone else puts his head down.

31. Roman Race 3-6

With paper and pencil, give the class 10 minutes to write the numbers from 1 to 100. Doesn't that sound easy? There is only one minor rule to this game. Please count in Roman numerals.

### 32. Roman Numeral Bingo 4-6

**Materials:** 4 bingo cards 9" x 9" per group. Divide cards into 16--2 1/4" squares. Print a different Roman numeral in each square. There should be 4 sets of 16 each 2" x 2" cover cards. Number words are written on one side and a Roman numeral on the reverse side to match the playing boards. Three sets are used by players, the fourth by the caller.

**Directions:** Each player takes a bingo card and a set of cover cards. The players spread their cover cards in front of them, Roman numeral side up. The caller draws a card from his stack, calls it and places it on his playing board. The other players find the card and place it on their board. This is played like Bingo. It is checked by looking at the answer side.

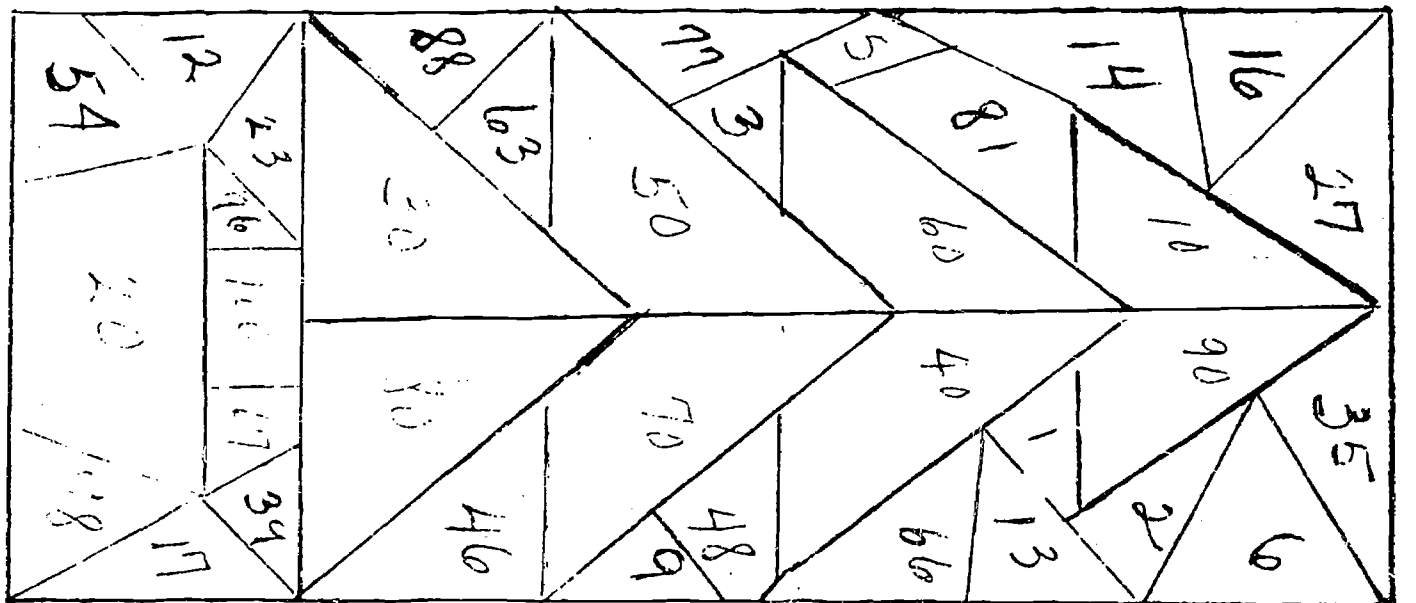
### 33. Roman Numeral Drill 2-3

On a printed worksheet have the class match the Roman numeral with the Arabic numeral.

1	IV
3	IX
2	V
6	X
4	VII

cont. 7	VIII
8	I
10	III
5	II
9	VI

### 34. Find the Hidden Object -- Shade in the ten's.



35. Who Am I? 1-2

One child is "IT". He comes to the front of the room and thinks of any number between 1-100. He gives a clue so the rest of the class can guess what it is. Suppose "IT" was thinking 53. He would say "I am 5 tens and 3 ones. Who am I?" The child who guesses correctly when called on is the new "IT". This could be used with groups around the room.

36. Number Board 1-2

**Materials:** One square y of plywood or lightweight wooden sheet in which 10 rows and 10 columns o ls have been pounded in.

**Directions:** I have a new game for you to play in your free time. In this box are 100 circles numbered from 1 to 100. To play this game, you hang numbers on a nail. When you finish, the numbers should be in proper order. (At first the teacher should check this--then have the children do it). This could be used for odd and even or skip counting.

37. Count to 100 1-2

Call on someone to begin counting to 100. After a short time say stop and call on another child and so on. Continue in this way until you reach 100. The child must listen carefully because he never knows when he may be called on.

38. Bad Egg 1-3

**Materials:** On a stencil sheet, scatter numbers from 1-100. Write the number twice. Duplicate one sheet for each child. Children will need a pencil.

**Directions:** On this sheet are the numbers from 1-100, but one number is written twice. Call that number the Bad Egg. To find this number circle all the numbers in sequence starting with 1. This could be used for skip counting by using 2's, 5's, 10's. The bad egg could be a number not needed. It could also be used for odd and even by putting all odd or all even numbers and the bad egg would be one of the opposite.

39. Automobile Number Game

**Materials:** 3 toy cars, 50 cards with the numbers from 1 to 50, a large number line on the floor that goes up to 50.

**Directions:** Children form three lines; the first one in each line draw a card from the teacher, reads the number, and moves his car that many places forward. The team getting to 50 is the winner. This game goes very fast so it could be played many times.

#### 40. Place Value Drill 2-6

Materials: 30 squares of tagboard numbered from 0-9, making 3 cards for each number. Put tens, ones, and hundreds on each set of 3.

Directions: I have given 30 of you cards with numbers on them. Look carefully at your number and at the word written below that number. Those of you without cards will call out numbers, such as 385. The ones that represent these numbers will come to the front of the room and arrange themselves in proper order. This could be done by giving 15 children two cards.

#### 41. Exponents

1. Make true statements.

(a)  $5^2 = 5 \times \underline{\hspace{1cm}}$       (d)  $\underline{\hspace{1cm}} = 125$       (g)  $\underline{\hspace{1cm}} = 15,625$   
- (b)  $5 \times 5 = \underline{\hspace{1cm}}$       (e)  $\underline{\hspace{1cm}}^4 = 625$       (h)  $\underline{\hspace{1cm}} = 5^7$   
(c)  $\underline{\hspace{1cm}}^3 = 5 \times 5 \times 5$       (f)  $\underline{\hspace{1cm}} = 5^5$       (i)  $5^1 = \underline{\hspace{1cm}}$

2. Make true statements.

- (a) The exponent in  $5^3$  means that 5 is used as a factor  $\underline{\hspace{1cm}}$  times.  
(b) Since  $125 = 5^3$ , we say that 5 is the  $\underline{\hspace{1cm}}$  power of 125.  
(c) The exponential form of  $5 \times 5 \times 5 \times 5$  is  $\underline{\hspace{1cm}}$ .

3. Name each of the following as a product of factors. Then write the standard numerals for the products.

(a)  $5^5 = \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$       (d)  $2^6 = \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$   
(b)  $1^7 = \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$       (e)  $10^4 = \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$   
(c)  $3^5 = \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$       (f)  $12^2 = \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$

4. Name each product in exponential form.

(a)  $6 \times 6 \times 6 = \underline{\hspace{2cm}}$       (d)  $10 \times 10 = \underline{\hspace{2cm}}$   
(b)  $8 \times 8 \times 8 \times 8 = \underline{\hspace{2cm}}$       (e)  $7 \times 7 \times 7 \times 7 \times 7 = \underline{\hspace{2cm}}$   
(c)  $12 \times 12 \times 12 \times 12 = \underline{\hspace{2cm}}$       (f)  $1 \times 1 \times 1 \times 1 \times 1 \times 1 = \underline{\hspace{2cm}}$

5. Write standard numerals for each of the following.

(a)  $2^3 = \underline{\hspace{1cm}}$       (c)  $5^3 = \underline{\hspace{1cm}}$       (e)  $7^2 = \underline{\hspace{1cm}}$       (g)  $11^4 = \underline{\hspace{1cm}}$   
(b)  $3^2 = \underline{\hspace{1cm}}$       (d)  $1^9 = \underline{\hspace{1cm}}$       (f)  $4^3 = \underline{\hspace{1cm}}$       (h)  $9^2 = \underline{\hspace{1cm}}$

continued

6. Make true statements.

- (a) 9 is the \_\_\_ power of 3  
(b) \_\_\_ is the 3rd power of 7  
(c) 4096 is the \_\_\_ power of 8  
(d) 36 is the 2nd power of \_\_\_  
(e) 729 is the \_\_\_ power of 9  
(f) \_\_\_ is the 2nd power of 11  
(g) \_\_\_ is the 5th power of 6  
(h) 100 is the \_\_\_ power of 10  
(i) 1 is the 12th power of \_\_\_  
(j) 1024 is the \_\_\_ power of 4  
(k) 64 is the \_\_\_ power of 8  
(l) \_\_\_ is the 3rd power of 1

7. Give standard numerals to make true statements.

- (a) The product of  $2^2$  and  $3^1$  is \_\_\_\_\_. (c) The product of  $5^2$  and  $2^3$  is \_\_\_\_\_.  
(b) The product of  $3^2$  and  $4^2$  is \_\_\_\_\_. (d) The product of  $3^3$  and  $5^1$  is \_\_\_\_\_.

42. Base-Five Numeration System

1. Group the dots pictured below by twenty-fives, fives and ones.

. . . . .  
. . . . .  
. . . . .  
. . . . .  
. . . . .

- (a) The base-ten numeral for the number of dots pictured is \_\_\_\_\_.  
(b) The base-five numeral for the number of dots pictured is \_\_\_\_\_.

2. Follow these steps to change  $88_{\text{ten}}$  to an equivalent base-five numeral. Think of a set of 88 objects.

- (a) How many subsets each containing twenty-five members can be formed?  
 $88 = (3 \times 25) + \underline{\hspace{2cm}}$   
(b) What numeral should be written in the twenty-fives place? \_\_\_\_\_  
(c) The difference of 88 and  $3 \times 25$  is \_\_\_\_\_.  
(d) How many subsets each containing five members can be formed from a set containing thirteen members? \_\_\_\_\_  $13 = (2 \times 5) + \underline{\hspace{2cm}}$   
(e) What numeral should be written in the fives place? \_\_\_\_\_  
(f) The difference of 13 and  $2 \times 5$  is \_\_\_\_\_.  
(g) What numeral should be written in the ones place? \_\_\_\_\_  $88_{\text{ten}} = 32_{\text{five}}$

3. Complete the table of numerals below.

Base-ten	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Base-five	0	1			4						20					31	

## 42. Continued

4. Tell if the number named is even or if it is odd.

(a)  $32_{\text{five}}$  \_\_\_\_\_

(f)  $221_{\text{five}}$  \_\_\_\_\_

(k)  $32_{\text{five}}$  \_\_\_\_\_

(b)  $103_{\text{five}}$  \_\_\_\_\_

(g)  $331_{\text{five}}$  \_\_\_\_\_

(l)  $102_{\text{five}}$  \_\_\_\_\_

(c)  $133_{\text{five}}$  \_\_\_\_\_

(h)  $40_{\text{five}}$  \_\_\_\_\_

(m)  $33_{\text{five}}$  \_\_\_\_\_

(d)  $41_{\text{five}}$  \_\_\_\_\_

(i)  $300_{\text{five}}$  \_\_\_\_\_

(n)  $21_{\text{five}}$  \_\_\_\_\_

(e)  $101_{\text{five}}$  \_\_\_\_\_

(j)  $21_{\text{five}}$  \_\_\_\_\_

(o)  $222_{\text{five}}$  \_\_\_\_\_

## 43. Addition in Base-five

1. We can easily complete the base-five addition table shown below by using what we have learned.

(a) What property of addition enables you to complete the base-five addition table without computing?

(b) Complete the table without computing.

(c) What other property of addition is illustrated in this table?

+	0	1	2	3	4
0	0	1	2	3	4
1			2	3	4
2				4	10
3					11
4					12

2. Make true statements and tell what property of addition was used.

(a)  $12_{\text{five}} + 31_{\text{five}} = \text{_____}_{\text{five}} + 12_{\text{five}}$

(b)  $\text{_____} + 14_{\text{five}} = 14_{\text{five}} \text{_____}$

(c)  $21_{\text{five}} + (\text{_____}_{\text{five}} + 30_{\text{five}}) = 21_{\text{five}} + (30_{\text{five}} + 23_{\text{five}}) \text{_____}$

(d)  $(31_{\text{five}} + 12_{\text{five}}) + 40_{\text{five}} = 31_{\text{five}} + (12_{\text{five}} + \text{_____}_{\text{five}}) \text{_____}$

(e)  $12_{\text{five}} + 0 = \text{_____}_{\text{five}} \text{_____}$

(f)  $(3 + \text{_____}) + 1 = 4 + (\text{_____} + 1) \text{_____}$



### 43. Continued

#### 3. Make true statements.

(a)  $4 + 1 = \underline{\hspace{1cm}}_{\text{five}}$

(e)  $4_{\text{five}} + 10_{\text{five}} = \underline{\hspace{1cm}}_{\text{five}}$

(b)  $44_{\text{five}} + 1 = \underline{\hspace{1cm}}_{\text{five}}$

(f)  $44_{\text{five}} + 10_{\text{five}} = \underline{\hspace{1cm}}_{\text{five}}$

(c)  $444_{\text{five}} + 1 = \underline{\hspace{1cm}}_{\text{five}}$

(g)  $444_{\text{five}} + 10_{\text{five}} = \underline{\hspace{1cm}}_{\text{five}}$

(d)  $4444_{\text{five}} + 1 = \underline{\hspace{1cm}}_{\text{five}}$

(h)  $4444_{\text{five}} + 10_{\text{five}} = \underline{\hspace{1cm}}_{\text{five}}$

#### 4. Name the sums as base-five numerals.

(a)  $\begin{array}{r} 1 \\ 2 \\ \hline 1 \end{array}$

(b)  $\begin{array}{r} 2 \\ 2 \\ \hline 2 \end{array}$

(c)  $\begin{array}{r} 3 \\ 2 \\ \hline 1 \end{array}$

(d)  $\begin{array}{r} 3 \\ 3 \\ \hline 3 \end{array}$

(e)  $\begin{array}{r} 4 \\ 3 \\ \hline 1 \end{array}$

(f)  $\begin{array}{r} 2 \\ 2 \\ \hline 3 \end{array}$

(g)  $\begin{array}{r} 13_{\text{five}} \\ 20_{\text{five}} \\ \hline \end{array}$

(h)  $\begin{array}{r} 34_{\text{five}} \\ 12_{\text{five}} \\ \hline \end{array}$

(i)  $\begin{array}{r} 44_{\text{five}} \\ 11_{\text{five}} \\ \hline \end{array}$

(j)  $\begin{array}{r} 40_{\text{five}} \\ 23_{\text{five}} \\ \hline \end{array}$

(k)  $\begin{array}{r} 43_{\text{five}} \\ 33_{\text{five}} \\ \hline \end{array}$

(l)  $\begin{array}{r} 41_{\text{five}} \\ 30_{\text{five}} \\ \hline \end{array}$

(m)  $\begin{array}{r} 31_{\text{five}} \\ 12_{\text{five}} \\ \hline \end{array}$

(n)  $\begin{array}{r} 40_{\text{five}} \\ 14_{\text{five}} \\ \hline \end{array}$

(o)  $\begin{array}{r} 30_{\text{five}} \\ 34_{\text{five}} \\ \hline \end{array}$

(p)  $\begin{array}{r} 41_{\text{five}} \\ 41_{\text{five}} \\ \hline \end{array}$

44. Make a calendar using other bases besides base 10. Example below taken from a 1967 calendar.

JANUARY 30332 <sub>five</sub>							
Sun	Mon	Tue	Wed	Thu	Fri	Sat	
1	2	3	4	10	11	12	
13	14	20	21	22	23	24	
30	31	32	33	34	40	41	
42	43	44	100	101	102	103	
104	110	111					

#### 45. Prime and Composite Numbers

"Prime" (Arithmetic Teacher, February, 1969)

This game is similar to Bingo. On a ditto construct a five inch square, then divide it into twenty-five inch squares. Above the five columns print the letters, P-R-I-M-E. Select sets of twenty-five numbers between 1 and 99 from a table of random numbers. Place one number in each square on each game sheet.

On slips of paper, write or type an appropriate selection of the following statements for each of the letters P-R-I-M-E. (In items 21 and 22, you may select the number to take the place of the blank)

1. A prime number
2. The largest prime number
3. The smallest prime number
4. A composite number
5. The largest composite number
6. The smallest composite number
7. A twin prime
8. A composite number between a pair of twin primes
9. A single-digit prime number
10. A two-digit prime number
11. A two-digit composite number
12. A two-digit number if either of its digits is a prime number
13. A two-digit number, the sum of whose digits is a composite number
14. A two-digit number with a prime number in the ones place
15. A two-digit number, the sum of whose digits is a prime number
16. A two-digit number, with a composite number in the ones place
17. A two-digit number with a prime number in the tens place
18. A two-digit number with a composite number in the tens place
19. A two-digit composite number with both of its digits prime numbers
20. A prime number less than \_\_\_\_\_
21. A prime number greater than \_\_\_\_\_
22. An even prime number
23. An odd prime number

Your rule slips should look something like this:

M: A two-digit prime number

Pass out the game sheets to the students. Spread out the rule slips face down in front of you. Choose at random a rule and read it aloud to the students. If a student has a number on his game sheet, under the appropriate letter P, R, I, M, E, that is defined by the rule, he is to put an X in that square.

For checking purposes, have the students write the number of the rule in the lower right-hand corner of the square in which he places the X. The first rule read would be considered number one; the second rule read would be considered number two, and so on. Keep these rules in the order that they are read.

Make sure that the students understand that only one square can be crossed out per rule, and that after a square is crossed out, the number in it cannot be considered for any following rules.

#### 45. Continued

Continue selecting rules and reading them aloud to the students until someone has crossed out all the blocks in a single column, row, or diagonal. Have the winner read aloud the numbers in the row, column, or diagonal, along with the rule that permitted him to cross out each of the numbers. Record these rules and numbers on the chalkboard and then have the class help you to check the validity of the winner's card.

The ditto may look as follows:

P	R	I	M	E
47	13	54	77	38
72	38	16	11	5
28	46	9	76	23
27	62	61	99	41
30	97	54	20	39

## NUMERATION AND NUMBER THEORY

### FILMSTRIPS

Eye Gate House, Inc.  
146-01 Archer Ave.  
Jamaica, N. Y. 11435

\$5.00 for each filmstrip

Set I--103A Counting  
103B Numbers 1-5  
103C Numbers 6-10  
103D Groups of 2-10  
Set II-106B Counting by 1's, 2's, and 5's

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Herbert M. Elkins Co.  
10031 Commerce Ave.  
Tujunga, California 91042

Set 84--No. 635 Numberland  
No. 636 Numbers for Beginners  
No. 637 Building 10's and 1's

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Society for Visual Education, Inc.  
1345 Diversey Parkway  
Chicago, Illinois 60614

532-8 Numeration: Base 10  
532-16 Numeration: Base 5  
532-21 Numeration: Base 6  
532-23 Numeration: Binary

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### FILMS

University of South Dakota  
Film Library  
Extension Division  
Vermillion, South Dakota

Primary: "Let's Count" - ordinal, cardinal, 11 minutes  
"Ones, Tens, Hundreds" - Place Value, 11 minutes

Intermediate: "Story of Our Number System" - 11 minutes

## NUMERATION AND NUMBER THEORY

### TRANSPARENCIES

Colburns  
2702 Montana Ave.  
Billings, Montana 59101

Mathematics Readiness Series 1400--14 Transparencies with overlays plus 80 cut-out, multi-colored. Includes: Numerals and Their Names; Writing Numerals; and Counting by 2's-5's.

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Stanley Bowman Co., Inc.  
4 Broadway  
Valhalla, N. Y. 10595

Number Concepts 1-10--Set 510016x--transparencies with flay (10) - \$40.00

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### RECORDS

Learning Arts  
P.O. Box 917  
Wichita, Kansas 67201

Counting Games and Rhythms for Little Ones - \$4.15

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American Encyclopedia of Learning through Music

All About the Numbers and Counting

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Stanley Bowman Co., Inc.  
4 Broadway  
Valhalla, N. Y. 10595

30.5000 2-12 Long play - Teaching children mathematics through games, rhythms and stunts. Special Education and Kdgn.

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### GAMES

Learning Arts  
P.O. Box 917  
Wichita, Kansas 67201

Games for Growth in Math  
Books 1 and 2 - \$.98 When bought in set of 10 or more

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## GAMES (CONT.)

Stanley Bowman Co., Inc.  
4 Broadway  
Valhalla, N. Y. 10595

Games for Growth in Math - Primary

99.27 Book 1 - \$1.50

99.28 Book 2 - \$1.50

-----  
School Service Company  
4510 W Pico Blvd.  
Los Angeles 6, California

Commercial Games for Grades K-2

Bingo - No. 5610 - \$1.00

-----  
O. Schoenut, Inc.  
Philadelphia, Penn.

Colored Pick-up Sticks (primary) - \$.49

-----  
Math Media Division  
N & M Associates,  
P.O. Box 1107  
Danbury, Conn. 06810

M102 Doalie Bobbers K-Up

Small colored notched discs which interlock. Can build 3-dimensional designs

M103 Lego Jumbo Brick Set

60 jumbo size 1" x 2" x 4" in red, white and blue. Stimulates creative activity

M107 Construct-o-straws ~ Grade 1-5

Consists of polyethylene tubes and an assortment of 8 different kinds of plastic joiners. Develops creative imagination, manual dexterity, and understanding of basic concepts.

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## ADDITIONAL ADDRESSES

Herder & Herder  
232 Madison Ave.  
New York, N. Y. 10016

WFF'N Proof  
Box 71  
New Haven, Conn. 06501

Study-Scope  
319 E Front St.  
Tyler, Texas 75701

Math Media Division  
N & M Associates  
P.O. Box 1107  
Danbury, Conn. 06810

Selective Educ. Equip.  
Inc.  
Three Bridge Street  
Newton, Mass. 02195

Science Research  
Ass., Inc.  
259 East Erie St.  
Chicago, Ill. 60611

INTRODUCTION TO ADDITION EQUATIONS:  
COMBINATIONS THROUGH 5

III. REAL NUMBER  
OPERATION  
Addition and  
Subtraction.

1. S

1. Given a page of addition combinations to 5 in equation vertical notation the pupil should be able to write the sums or missing addends in 5 minutes.

Readiness. Teacher evaluation of child's understanding of the union of sets.

- 1a Addison Wesley Book 1, pages 57 - 65
- b. Duplicator Masters Book 1, pages 19-22
- c. Teacher's Manual 1968 edition.
- d. Number line on the floor made of light colored "Contact" split down the center.
- e. See activities 1 - 13.
- f. Cyclo-Teacher M-4
- g. Evaluation: Teacher made test pages 68, 70 test.

SUBTRACTION EQUATIONS  
COMBINATIONS THROUGH 5

III. REAL NUMBER  
OPERATIONS  
Addition and  
Subtraction.  
G. 1. S

2. Given a page of subtraction combinations through five, expressed in equation or vertical form, the child will supply the difference in 5 minutes.

Readiness: Satisfactory completion of a test over the addition facts 1-5.

3. Given a page of sets picturing combinations to 5 the child will write 4 equations for each showing the inverse relationship between addition and subtraction with 85% accuracy.

- 2a. Addison Wesley, Book one pages 73-96.

- b. Duplicator Masters 23-31

- c. Invent stories to involve children in acting out subtraction equations. (Walk away, sit down, etc).

- d. Number Puzzles  
(teacher's Manual A-W good)

- e. Cyclo-teacher 11-13

- f. See activities 1-13

- g. Evaluation: Teacher-made test.



ADDITION AND SUBTRACTION EQUATIONS  
COMBINATIONS THROUGH 9

III. REAL NUMBER  
OPERATIONS  
Addition and  
Subtraction.

G. 1. S

4. The pupil should be able to write the sums or differences in a mastery test covering all combinations of addition and subtraction through 9 with 95% accuracy in four minutes.

Readiness:

Test of child's ability with combinations to 5.

- 4a. Addison Wesley, Book 1 97-152
- b. Teacher's Edition 1968 for activities
- c. Duplicator masters 30-41
- d. Cyclo-teacher M-5,6,7,8,9,10  
Addition Subtraction M 14-19
- e. Use stories to involve child activity with coins (real or play.)
- f. See activities 1-13, 16.
- g. Evaluation:  
Addison Wesley 144, 149, 151, 152  
Teacher-made test.

ADDITION AND SUBTRACTION  
THROUGH 10

III. REAL NUMBER  
OPERATIONS  
Addition and  
Subtraction

1. 1
2. R, S

5. Given a mastery test in subtraction and/or addition with all combinations through 10, the learner should be able to write the differences or sums within 3 minutes with 95% accuracy.

Readiness: Teacher's observation of child's understanding of order and grouping principles and place value.

- 5a. Book 1 - Addison Wesley, pg. 199-222.
- b. Teacher's for activities, pg. 238-261
- c. Duplicator Master's pg. 55-62
- d. Book 2 - Addison Wesley, pg 53-70
- e. Teacher's Edition 1968, pg 84-103
- f. Duplicator Masters related
- g. See Activities 1-17, 28, 30.

Evaluation;

A. W. pg. 208, 213, 217, 218,  
Cumulative 221-222.

III. REAL NUMBERS  
OPERATIONS

Addition

G. 1. S

PROPERTIES  
ORDER (COMUTATIVE)  
GROUPING (ASSOCIATION)

6. Given problems such as  $28+34=$   
 $34+ \underline{\hspace{1cm}}$  and  $12+(6+11) = (\underline{\hspace{1cm}} + \underline{\hspace{1cm}})$   
 $+11$ , the learner will be able  
to demonstrate his understanding  
of the order and grouping prin-  
ciples by completing the pat-  
terns with 85% accuracy.

Readiness:

Teacher observation.

6a. Addison Wesley, Book 1, pg 223-234.

b. Teacher's Second Edition pg. 262-  
275.

c. Duplicator Masters 63-65

d. Arithmetic Teacher, Nov. 1965  
"Basic Laws for Young Children."

e. See Activity 24.

III. REAL NUMBERS  
OPERATIONS

Addition

1 -1.

2. R and S

ADDITION COMBINATIONS  
THROUGH 18

7. Given a test in addition com-  
binations through 18 and a  
choice of methods, the pupil  
should be able to compute the  
sums with 85% accuracy.

Readiness. Teacher observation of  
child's understanding of group-  
ing and order principles and  
place value.

7a. Book 1, Addison Wesley, 235-258

b. Teacher's second edition, 276-301

c. Duplicator Masters, 66-73

d. Book II Addison Wesley, 71-96

e. Teacher's edition 1960, 104-131

f. Duplicator Masters as they relate.

g. See Activities 7-17, 25-30.

ADDITION AND SUBTRACTION  
THROUGH 18

III. REAL NUMBERS  
OPERATIONS  
Addition and  
Subtraction

- G. 1. I  
G. 2. S  
G. 3. R  
G. 4. R  
G. 5. R

8. Given a test in addition and subtraction combinations through 18 and a choice of method, the pupil will compute the sums or differences with 85% accuracy.

Readiness. Test over facts to 10.

8. a. Book 1, Addison Wesley 1963 pg. 259-268  
b. Duplicator Masters 74-75  
c. Book 2 A. W. 1963, 97-140  
d. Book 2, Duplicator Masters pages related.  
e. Book 3, 1963, A. W. 48-77  
f. Teacher's edition 1968, 20-43  
g. Related Duplicator Masters.  
h. Workbook to accompany above.  
i. Book 4 A. W. pg. 48-55 1963  
j. Teacher's edition 1968 21-27  
k. Related Duplicator Masters.  
l. Book 5, 1963, 20-23, 30-31  
m. Teacher's edition 1968, 18-45 Some activities that can be adapted.  
n. Omit digits from the problems  
+ 4            7            9            ?  
— 9        — 5        — 2            7  
12        5                14  
o. See activities 7-17, 25-30

ADD OR SUBTRACT THREE TO FOUR  
DIGIT NUMBERS (NO REGROUPING)

III. REAL NUMBERS  
OPERATIONS.  
Addition and  
Subtraction.

G. 2. S  
G. 3. R

9. Given ten addition and ten subtraction examples having pairs of numerals with three or four digits and guided by operational signs, the pupil will write the sums or differences without regrouping. Successful performance is 85% accuracy in 20 minutes.

Readiness: Successful completion of a test over combinations to 10.

- 9.a. Addison Wesley Book 2, 1963 edition. pg. 199-208.
- b. Teacher's Manual pp. 241-251
- c. Duplicate Masters, Book 2.
- d. A. W., Book 3, 1963, pp 78-95
- e. A. W., Book 3 Teacher's Manual pp. 51a-67.
- f. See activities 27, 31, 35-38, 41.
- g. Evaluation: Teacher-made test or may use pp 199, 201, 202, 208 from text.

ADDITION: TWO DIGIT NUMBERS  
WITH REGROUPING

III. REAL NUMBERS  
OPERATIONS  
Addition and  
Subtraction

- G. 2. S  
G. 3. R  
G. 4. R, E.

10. Given a page of two-digit addition examples requiring regrouping, the learner will write the sums with 85% accuracy.

Readiness: Test of understanding of regrouping with sets.

10. a. Addison Wesley, Book 2, 1968

b. Duplicator Masters, 1969

c. A. W. Book 3 pp 98-100

d. Duplicator Masters.

e. A. W. Book 4, 1963, 40-63

f. Duplicator Masters.

g. Used expanded notation and renaming.

$$\begin{array}{r} 51 = 50 + 1 = 40 + 11 \\ - 24 = 20 + 4 = \frac{20 + 4}{20 + 7} = 27 \end{array}$$

h. The Cyclo-teacher M12, M21.

i. See activities 18-23, 27, 29, 30, 35, 37, 39, 40, 41

j. Evaluation: Teacher test of child's masters of the standard addition algorithm.  
Teacher observation of child's understanding of regrouping.

SUBTRACTION: TWO DIGIT NUMBERS  
WITH REGROUPING

III. REAL NUMBERS  
OPERATIONS  
Addition and  
Subtraction

G.2. S  
G 3. R

11. Given ten subtraction examples having pairs of two-digit numerals, the pupil will use regrouping to write the differences with 85% accuracy.

Readiness : Teacher observation of child's ability to use sets of objects in the regrouping process. The thought process is the important item to be checked.

11. a. Addison Wesley Book 2, 1968, pp 221-240  
b. Teacher's Edition 1968, pp 226-287  
c. Duplicator Masters 1969  
d. Book 3, 1963 pp 101-110  
e. Teacher's edition 1968, pp 20-43  
f. See activities 27, 35, 38.  
g. Daily observation of understanding of regrouping. Test of computation efficiency. Text p. 239-- a cumulative review.

ADDITION AND SUBTRACTION  
THREE DIGIT NUMBERS  
WITH REGROUPING.

III. REAL NUMBER  
OPERATIONS  
Addition and  
subtraction

G. 3. S  
G. 4. R

12. Given a page of addition examples, each having columns of three-digit numerals, the learner can write the sums with 85% accuracy.

Readiness: Test child's understandings of regrouping involving tens and place value.

13. Given a page of subtraction examples having pairs of three-digit numerals, the learner will write the differences with 85% accuracy. Each example will call for regrouping from the hundreds place and some should have zero in the ones place.

12. a. Addison Wesley Book 3, 1963  
pp. 108-111

b. Teacher's Edition 1968, pp. 51-83

c. Duplicator Masters , 1969 Book 3.

d. Addison Wesley Book 4, 1963  
pp. 64, 77.

e. Teacher's Edition 1968, pp. 22-55

f. Duplicator Masters 1969

g. "Practice in Renaming Numbers--  
An Aid to Subtraction" --The  
Arithmetic Teacher, Feb. 1965.

h. See activities 20-23, 27, 31, 35,  
37, 39, 40, 42, 43

i. Evaluation. Teacher-prepared  
tests.

Teacher observation of child's  
ability to think and reason.

Text Pp. 82-83



ADDITION AND SUBTRACTION  
FOUR-DIGIT NUMERALS  
WITH REGROUPING

III. REAL NUMBERS  
OPERATIONS  
Addition and  
Subtraction.

G. 4. S  
G. 5. R  
G. 6. R, E

14. Given a set of ten examples such as:

$$\begin{array}{r} 896 \\ 784 \\ 599 \\ 867 \\ \hline 657 \end{array} \quad \begin{array}{r} 6785 \\ + 8432 \\ \hline \end{array} \quad \begin{array}{r} 8000 \\ - 6385 \\ \hline \end{array}$$

the learner can write the sums or differences with 90% accuracy within 20 minutes.

- 14.a. Addison Wesley, Book 4 pp 64-69,  
b. A. W. Teacher's Edition 1968 pp. 22-63  
c. Duplicator Masters 1969  
d. A. W. Book 5, pp. 86-95  
e. Teacher's edition 1968, pp . 28-29. 44-45. 102-111.  
f. Duplicator Masters 1969  
g. Addison-Wesley Book 6 pp. 44-49  
h. Duplicator Masters 1969  
i. See Activities 20, 22, 27, 31, 35, 37, 39, 40, 42.  
j. Evaluation: Teacher-made test and Chapter Review.

## ACTIVITIES

1. A clothespin (or bead) line can be made from a wire coat hanger and spring-type clothespin. Add pins as you work with more difficult numbers. It can be used to make a transition from horizontal to vertical form in addition and subtraction.

### 2. Spin It.

Materials. Oaktag, paper clip, brass paper fastener.

Directions. Draw a 6" circle on a square piece of tagboard. Divide that circle into twelve equal sections. In each section write an addition or subtraction fact. Use the brad to fasten the paper clip to the center of the circle. To play the game each child spins the clip in turn, reads the number fact to which it points, and responds with the sum or difference. The child with the most correct answers wins.

### 3. Other names for numerals.

---

0 + 0	2 + 0
0 - 0	1 + 1
1 - 1	0 + 2
2 + 2	2 - 0
3 - 3	3 - 1
4 - 4	4 - 2

### 4. Addition Combination Chart. ( 1-10 )

1	2	3	10
1 + 0	2 + 0	3 + 0	10 + 0
0 + 1	1 + 1	2 + 1	9 + 1
	0 + 2	1 + 2	8 + 2
		0 + 3	7 + 3
			6 + 4
			5 + 5
			4 + 6
			3 + 7
			2 + 8
			1 + 9
			0 + 10

Subtraction table in same manner.

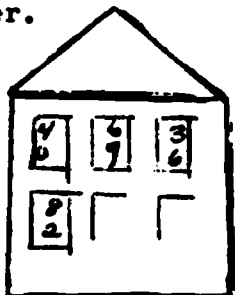
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8
10 - 2
9 - 1
8 - 0

## 5. Number House Grade 1 -2

Materials: Number houses  $8\frac{1}{2}'' \times 5\frac{1}{2}''$  either teacher made or made by the students. The houses should be made of heavy paper or oaktag doubled to open like a book (see example). Cut the windows out in order to see the answer.

Example:

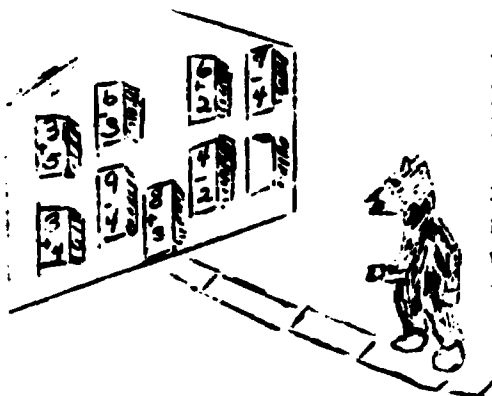


Directions: Have two children quiz each other on either subtraction or addition facts. Check your answer by opening the window. The children could trade houses. They could also play a game to see who could win the most points by being correct.

## 6. Numbers and Stories.

Write three numerals such as 3, 7, 2, on the chalkboard. Ask the class to tell a story about these three figures. Example; "Two children went walking. Five other children joined them, so there were seven children walking along together. Four children stopped to play so there were only three walking."

### 6 A. Red Riding Hood and the Wolf



Could be used on a bulletin board or flannel board. Make the house of tagboard or felt. Place number facts in the windows and door. Leader points to a fact. If player responds correctly the shutter is closed. If response is incorrect the cutout wolf moves one step nearer. The game is to see if all the windows and the door can be closed before the wolf reaches the house.

## 7. Secret Code

The leader taps out an "addition" message. (Could use triangle from rhythm instruments). A child at the board records the message in column (or equation) form and computes the sum.

## 8. Roll the Hoop

Write number combinations in addition or subtraction on the spokes of this wheel. The player makes the wheel turn as fast as he can by giving the answers in order.



## 9. Hide and Seek

Write combinations on the board with the different parts missing, as  $5 + 2 + \underline{\quad}$ ,  $7 - \underline{\quad} = 2$  etc. The child should fill in the blanks and repeat the combinations.

## 10. In and Out.

Use large flash cards showing the facts on which you are working. The pupils stand in a circle. Going around the circle, the child who gives a wrong answer must step within the circle while the next child answers. Should someone make an error, the child in the center is allowed to answer and resume his place in the circle.

## 11. Crossing the Bridge.

Flash cards showing combinations are placed on the floor at well-spaced intervals. The child pretends that each flash card is a plank in a bridge. In order to cross the bridge he must give the correct answer for each "plank" as he takes a step. If he makes a mistake he slips off and gets his feet wet.

## 12. Footprints.

Draw an igloo ( or space ship, tepee, Santa's workshop) and snowshoes or footprints on the board. Write combinations on the snowshoes. Each child has a chance to see if he can answer all the combinations. If he can, he can go to the igloo, spaceship etc. Change combinations each time.

## 13. Maintaining Facts.

Give each student a number card suitable to facts being studied. The teacher asks questions such as:

"I have five (holding up five card), Who can make it eight?"

The child with the three card comes up to join her.

"I have ten. Who can make it three?"

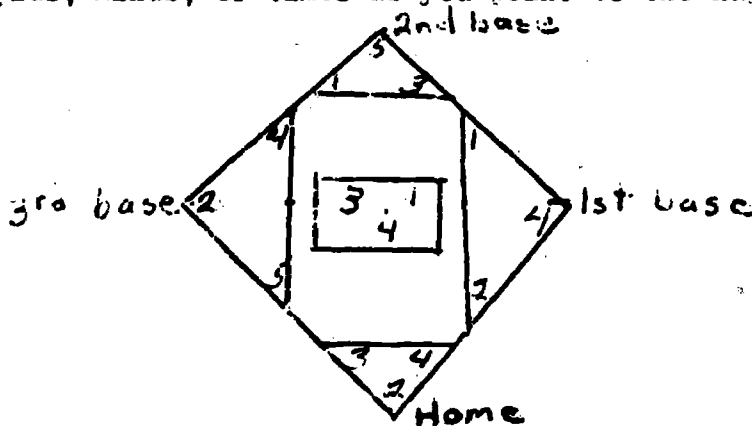
"I have seven. Who can make it 56?"

"I have 81. Who can make it 9?"

4	1	7
8		0
5	3	6

- cy: 1. 12 3 7 5. 5 7. 4 9. 1  
2. 2 4. 6 6. 13 8. 4 10. 3

Write several numerals at each base and in the pitcher's box as shown below. Select the numbers so that the sum of the numbers will not be greater than has been developed at that particular time. Two children act as captains and choose teams. The teacher points to one of the numerals in the pitcher's box and to one of the numerals at first base. The child who is at bat must give the correct sum for the numbers named. If he fails he is out. If he gives the correct sum, he keeps "batting". The teacher points to a number in the pitcher's box and one at second base. The play continues until batter is out or has scored a run. When three batters on one side are out, the other side takes its turn. Play continues for any desired number of innings. The side with more runs at the end of the game is the winner. Vary by saying plus, minus, or times as you point to the numbers.



16. Concentration      Grades 2, 3, 4, 5, 6

Introduction: Watch and listen to my rhythm pattern. When you are ready join me. The pattern is clap twice, snap finger right, then left twice. These four beats should be steady.

Now I will continue to clap and snap giving a problem. The first child in the row will clap as usual and as he snaps his fingers answer the problem. Then I will go to the next child. Let's see how far we can go before someone breaks the rhythm.

Example. Clap clap five - two

Clap clap se - ven

Could be done with multiplication.

17. Magic Squares - 3 x 3

$n-1$	$n-2$	$n+3$
$n+4$	$n$	$n-4$
$n-3$	$n+2$	$n+1$

18. How to construct Magic Squares.

- The number of small squares within a large square must be uneven-- 9, 25, 49, and so on.
- The first numeral is placed in the center small square of top row.
- To find the second square, move one square up and to the right. This takes you outside of the square, so go to the bottom of the next row.
- For the third square move up and to the right. If you are constructing a small magic square of 9 squares, this takes you outside the large square, so go to the far end of the horizontal row.
- For the 4th square drop to the square below. For squares 5 and 6 move up diagonally. This takes you back to the upper right hand square.
- Drop down for number 7.
- Number 8 is reached by jumping to to the left hand upper corner square. This leaves only the center square in the bottom row to be filled by 9.

18. Cont'd.

8	1	6
3	5	7
4	9	2

8	1	6
3	5	7
4	9	2

Interesting facts. The center square in either square (3 or 5). The Magic Square is the average of the sum or difference of the square.

### 19. Special Magic Squares.

Super Magic Square (Albrecht Durer)

4 squares. There are at least 40 different ways of finding groups of 34 in the square.

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

16	2	3	13
5	11	10	8
9	7	6	12
4	14	15	1

		3	13
		10	8

Reverse the

and line numbers.

Add the colored squares using the same numbers as in the original the original square. What is the answer for each grouping.

### 20. Magic Square #2

Only one of these three squares is a magic square, that is, a square in which the sum of the numbers named in each row, column, and diagonal is the same. Find the magic square.

a

82	287	246
369	205	123
164	41	328

b

328	41	246
123	205	287
164	369	82

c

164	369	82
123	205	41
328	287	246

d


Add 25 to each number in the square, and write the numerals for the sums in the empty spaces of square D. Prove that your new square is also a magic square. (Magic number is 640.)

### 21. Magic Square #3

Complete these magic squares.

Ⓐ	17	16	6
14	Ⓒ	7	11
10	12	Ⓔ	5
15	3	4	Ⓡ

$a=1$   
 $c=8$   
 $e=13$   
 $g=18$

101	Ⓐ	110	115
Ⓒ	113	103	106
207	102	16	Ⓔ
14	111	Ⓡ	104

$a=8$   
 $c=12$   
 $e=9$   
 $g=105$

34	48	2	16	Ⓐ
46	10	14	Ⓒ	32
Ⓔ	12	26	40	44
20	Ⓡ	38	42	6
22	36	Ⓡ	4	18

$a=30$   
 $c=28$   
 $e=8$   
 $g=24$   
 $r=50$

17	24	1	Ⓐ	15
Ⓒ	5	7	14	Ⓡ
4	Ⓡ	13	20	Ⓔ
Ⓐ	12	Ⓒ	21	3
11	18	25	6	Ⓔ

$a=8$   
 $e=23$   
 $h=16$   
 $f=6$   
 $g=22$   
 $d=10$   
 $c=11$   
 $b=2$

### 22. Magic Square #4

Once having completed a magic square, subtract 2 from each number. Find the sum of each column, row, and diagonal. (A new magic square.)

In a 3 x 3 magic square the center number is the average of the sum of the diagonals, or 5 times the number named in center.

In a 5 x 5 magic square each diagonal sum is 5 times as great as the center number.

### 23. Magic squares may be made in various ways.

- Consecutive numbers.
- By adding a certain number to the starting number.
- By subtracting a certain number from the starting number.



23. Cont'd.

Rule a

10	3	8
5	7	9
6	11	4

Rule a

$\frac{8}{4}$	$\frac{1}{4}$	$\frac{6}{4}$
$\frac{3}{4}$	$\frac{5}{4}$	$\frac{7}{4}$
$\frac{4}{4}$	$\frac{9}{4}$	$\frac{2}{4}$

Rule b  
+25

251	76	201
126	176	226
151	276	101

Rule C  
-12

63	147	87
123	99	75
111	51	135

d

17	24	1	8	15
23	5	7	14	16
4	6	13	20	22
10	12	19	21	3
11	18	25	2	9

#### 24. Properties in Addition or Multiplication--Associative and Commutative.

Give three children cards with numerals representing numbers no greater than nine. Have them stand in a straight line in front of the class. Have numbers read from left to right and the related equation written on the chalkboard and read aloud. The children will change places in accordance with arrangements suggested by the class.

#### 25. Slide Rule. For addition and subtraction to 18.

Use a piece of tagboard  $4\frac{1}{2} \times 20$ ". Fold in half lengthwise to make the body of a slide rule. With a felt pen, write the numerals 0 through 18 at intervals of one inch, one-eighth inch down from the top. Use another piece of tagboard  $3\frac{1}{2} \times 20$ " and write the numerals 0 through 18, two and one-half inches from the bottom of the tagboard. This is the "slide."

Place the slide in the body. Move the slide to the right until the zero is above one addend. Look along the slide for the second addend. On the body below the second addend is the sum.

	8	10	12	3	4	5	6	7	8	11	11	
A	0	1	2	3	4	5	6	7	8	9	10	11

Rule shows

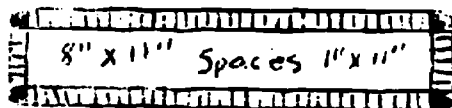
$3 + 2 = 5$   
 $3 + 3 = 6$   
 $3 + 5 = 8$   
 $3 + 8 = 11$  etc.

See May 1966 Arithmetic Teacher for suggestions--pp 403-404 for rulers to add, subtract, multiply and divide whole numbers.

## 26. First Back

Materials: A playing board as illustrated below. A set of playing cards such as  $1 + 2$ ,  $8 - 5$ ,  $4 + 4$  etc. piled face down in the center of the board. A marker for each player made out of a button or a piece of different colored paper.

Directions: Each child puts his marker on one of the colored spaces. One child draws a card. He reads aloud the phrase that is written on the card and gives the name of the number. If his response is correct he moves the marker a corresponding number of spaces in direction indicated in the illustration. If his answer is incorrect, he is not allowed to move his marker and the next child takes a turn. The player who goes all the way around and returns to his starting place first is the winner.

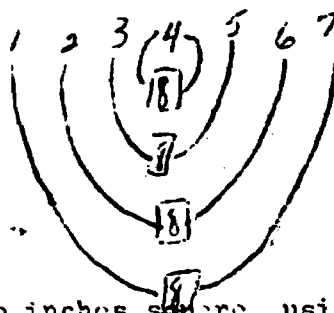
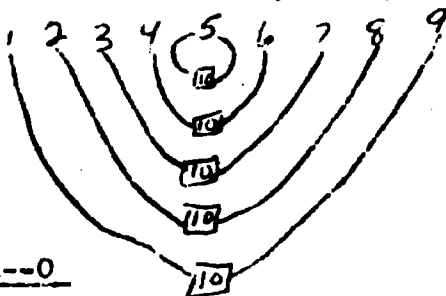


Each corner a different color

## 27. Relay Race.

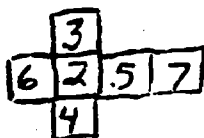
Separate class into 5-8 teams. Provide each team with a worksheet on which are written the basic facts studied so far. The first player completes the first exercise and hands the sheet on to the next player. The second player completes the second exercise, passes the sheet to the third player, and so on. Continue until all exercises are complete. Responses, once written, cannot be changed. Greatest number of responses correct wins.

28. Using Patterns for adding pairs of numbers--lead to appreciation of our orderly numeration system and experiment with patterns of own making.



## 29. Sum--0

Have each child take two blocks each, two inches square, using the pattern given. Write a number on each square with crayon before folding the oaktag into a block. Close the blocks with tape. Players toss the blocks and add the numbers turned up. (Could use sugar cubes or dice.)



## 30. Can You Match Me?

Two or three students of any age or ability can play independently. Use chalkboard or small objects in groups. One student composes a mathematical expression which can be matched in a number of ways by other students. Given:  $\frac{5 + 16}{3}$  Matching  $10 - 3$ ,  $15 - 8$ ,  $30 \div 3$

### 31. Progressive Addition

Each student writes an addition (or multiplication) combination such as  $9 + 6$  on a sheet of paper. This sheet is then given to the next person, who writes the answer to the first combination and adds another number to be added. The papers are passed again and again with each student adding the combination and writing another numeral to be added. Continue until the paper returns to the person whose name appears on the page, or the time expires. The owner must check for any errors.

$$\begin{array}{r} 9 \\ 6 \\ \hline 15 \end{array} \rightarrow \begin{array}{r} 15 \\ 7 \\ \hline 22 \end{array} \rightarrow \begin{array}{r} 22 \\ 9 \\ \hline 31 \end{array}$$

### 32. Calendar Fun.

All the sums are the same--54  
Use for arrow arithmetic.

10	11	12
17	18	19
24	25	26

$$10 \rightarrow \downarrow = 18$$

$$19 \swarrow \nearrow = 19$$

### 33. Calendar Fun #2

Select any block of 16 numerals--4 columns or 4 numerals each. Draw a box around any numeral and cross out the other numerals in that row and column. Select another numeral, draw a box around it, and cross out all the other numerals in that box and column. Repeat the step, and then draw a box around the remaining numerals. The sum of the numbers represented in the boxes will always be equal to the sum of the numbers represented in four corners or double the sum of numbers represented in opposite corners.

### 34. Calendar Fun #3

Ask children to find the sum of the numbers in a given cylinder block. Teacher writes the sum and covers it. The children will think the teacher is brilliant when he arrives at the answer so quickly. (The center number times the number of numbers in the array will give the sum of all the numbers included.)

### 35. Addition on a Grid. May also be used for multiplication.

69	46	23
76	18	58

→ 6481

201	103	98
237	159	78

← → 181

↓ ↓

6	8
14	8

14  
20

3	(5)
(4)	7

15  
28  
12 35 420

### 36. Using Tens in Addition.

$$\begin{array}{r} 8 \\ 4 \\ 7 \\ 2 \\ 6 \\ 3 \\ \hline 35 \end{array}$$
 Starting at the base of the column and adding upward.  $8 + 6 = 10$   
 $+ 4$ . Strike out the 6 because it was the last digit used to  
 obtain 10. Start with the 4 remaining after the ten has been  
 subtracted.  $4 + 2 + 7 = 10 + 3$  Strike out the 7.  
 $3 + 4 + 8 = 10 + 5$  Strike out the 8.  
 Put 5 at the bottom of the column. Three tens  
 have been struck out so place 3 in tens place.

### 37. Several columns can be added by tens.

$$\begin{array}{r} 229 \\ 357 \\ 288 \\ 908 \\ 674 \\ 860 \\ 399 \\ 834 \\ \hline 4507 \end{array}$$
 Beginning with the ones column add up:  $4 + 9 = 10 + 3$ .  
 Strike out the 9 because it was the last digit used to  
 obtain a group of ten.  $3 + 0 + 4 + 8 = 10 + 5$   
 $5 + 6 = 10 + 1$ ,  $1 + 7 + 9 = 10 + 7$ . Write the 7 at  
 the bottom of the ones column. "Carry" the 4 tens marked  
 out to the tens column.  $4 + 3 + 9 = 10 + 6$ ,  $6 + 6 =$   
 $10 + 2$ ,  $2 + 7 + 0 + 8 = 10 + 7$   $7 + 5 = 10 + 2$   
 $2 + 8 = 10$ . Zero goes at the bottom of the tens column.  
 Five tens are carried to the hundreds column.  
 $5 + 8 = 10 + 3$   $3 + 3 + 8 = 10 + 4$   $4 + 6 = 10$   
 $9 + 2 = 10 + 1$   $1 + 3 + 2 = 6$ . The 6 goes to the bottom  
 of the hundreds column. The 4 tens marked out are carried  
 to the thousands place.

### 38. check for Subtraction.

If nine or one of its multiples is in the subtrahend,  
 the sum of the digits in the remainder will equal the sum of  
 the digits in the minuend. Example

$$\begin{array}{r} 72 \\ - 36 \\ \hline 36 \end{array} \quad \begin{array}{r} 7 + 2 = 9 \\ 3 + 6 = 9 \\ 3 + 6 = 9 \end{array}$$

### 39. Compute these examples.

$$\begin{array}{r} 568 \\ + 923 \\ \hline 1491 \end{array} \quad \begin{array}{r} 5091 \\ 2376 \\ \hline 3115 \end{array} \quad \begin{array}{r} 9580 \\ - 7777 \\ \hline 1803 \end{array}$$

### 40. Beat the Clock. In aid to addition and subtraction bridging.

Materials: 1 piece of cardboard 30" x 36".

Draw margins 5" from the top and 3" from the sides of the cardboard.  
 Draw vertical lines every 3" and 1½". Draw horizontal lines every 2½"  
 and 1½". Use a sharp blade to remove the 3" x 2½" blocks. Letter and  
 number the chart with felt pen.

Have the student place chart against a chalk board. Have him add  
 a certain number such as 9 and 12, to each printed number, writing  
 only the answers in each space (shown on chart). Set a time limit  
 to help eliminate finger counting.

Beat the Clock

16	61	43	37	90	36	15
<del>0</del>	<del>0</del>	<del>2</del>	<del>7</del>	<del>9</del>	<del>3</del>	<del>1</del>
18	42	29	74	23	47	35
<del>1</del>	<del>4</del>	<del>2</del>	<del>7</del>	<del>2</del>	<del>4</del>	<del>3</del>
14	32	85	58	46		
<del>1</del>	<del>3</del>	<del>8</del>	<del>5</del>	<del>4</del>		
64	17	81				
<del>6</del>	<del>1</del>	<del>8</del>				
25						
<del>2</del>						
<del>5</del>						

41. Find the sum of a series.  $2 + 4 + 6 + 8 + 10 + 12$

Add the first and sixth addend = 14 (2 + 12)

Add the second and fifth addends = 14 (4 + 10)

Add the third and fourth addends +  $\frac{14}{42}$  (6 + 8)

42. Check for Addition (Cast out nines).

6489	0
7349	5
2381	5
2515	4
18734	5
23 =	5

Add the numbers across in each figure. Discard any whole nines, writing the remainder to the right. 6489 adds to 27. 27 contains 3 nines with no remainders. Repeat the process for each figure including the sum. Add the remainders above the addition line (14) and cast out nines. 5 remains.

If the latter number coincides with the remainder obtained from the addition answer, the answer can be assumed to be correct.

### 43. Addition-Subtraction Activities

Row 1.	35	16	11	38	46	63	26	11	59
	<u>-23</u>	<u>+2</u>	<u>+10</u>	<u>-13</u>	<u>-28</u>	<u>-24</u>	<u>+17</u>	<u>+3</u>	<u>-42</u>

Row 2.	28	57	35	55	70	18	33	27	17
	<u>-9</u>	<u>-17</u>	<u>-18</u>	<u>-16</u>	<u>-28</u>	<u>+15</u>	<u>-19</u>	<u>+18</u>	<u>+18</u>

Row 3.	91	44	64	73	87	92	48	91	46
	<u>-28</u>	<u>-18</u>	<u>-39</u>	<u>-41</u>	<u>-30</u>	<u>-30</u>	<u>-24</u>	<u>-39</u>	<u>-19</u>

Row 4.	16	17	35	71	97	17	62	14	95
	<u>+8</u>	<u>+9</u>	<u>-17</u>	<u>-42</u>	<u>-11</u>	<u>+19</u>	<u>-34</u>	<u>+13</u>	<u>-32</u>

Row 5.	48	51	87	72	62	25	70	90	21
	<u>+18</u>	<u>+19</u>	<u>-26</u>	<u>-19</u>	<u>-19</u>	<u>-22</u>	<u>+13</u>	<u>-46</u>	<u>+29</u>

Here is a February picture puzzle. Can you work it? Write the answers to all the problems on this page and then follow these directions.

Row 1: Put a ring around your four largest answers.

Row 2: Put a ring around your three largest answers.

Row 3: Put a ring around your two smallest numbers.

Row 4: Put a ring around your two largest answers.

Row 5: Put a ring around the smallest answer.

Find your largest answer in Row 1. Now take a red crayon and going in a clockwise direction, draw a line to each of your circled answers.

What is the picture? Would you like to color it with crayons?

# RECOGNITION OF MULTIPLICATION FACTS

REAL NUMBER OPERATIONS  
Multiplication

2. S

## INSTRUCTIONAL OBJECTIVES

1. The learner can recognize relationships between equivalent sets, repeated addition, and skip counting. He should be able to show this recognition by completing a test of 15 problems similar to the following with 85% accuracy.

$$5 + 5 + 5 + 5 =$$

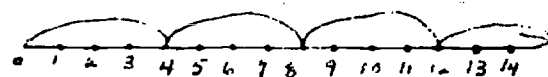
$$4 \times 5 =$$

0 0 0 0 0 0

0 0 0 0 0 0

$$3 \times 6$$

0 0 0 0 0 0



$$4 \times 4$$

## Activities.

- 1.a Text: Addison Wesley Grade 2 unit 13.

b Many games and activities are provided in the Teacher's manual 2nd Edition. A list of these games and the materials needed for them are listed on page 21 of this manual.

c. Old Scott Foresman paperback teacher's manual activity 13, P 337.

d Activities 1-6 in Multiplication Division activity section.

e Student authored problems. Students should be able to supply correct answers to their problems.

III REAL NUMBER  
OPERATIONS  
Multiplication and  
Division.

MULTIPLICATION AND DIVISION FACTS--  
INVERSE RELATIONSHIPS GROUPING PRINCIPLE  
DISTRIBUTIVE PRINCIPLE, REPEATED SUBTRACTION.

3. S

2. Given the set of multiplication facts whose products are 25 or less, the learner can write the answers with 90 accuracy in three minutes.
3. Given a group of division facts whose dividends are 25 or less, the learner can write the answers with 90% accuracy in three minutes.
4. Given a group of products less than 25, the learner can write the four-member multiplication-division fact team for each product with 85% accuracy.
5. Given a test of 10 word problems, the learner should solve them with 90% accuracy. These should be in the same difficulty range as the fact problems above.

Tests may be made from "Keeping in Touch" pages and sets 30-52 of the text. Grade 3  
P. 359 - 365.

- 2, 3, 4, 5.
- a. Addison Wesley Text--Grade 3; units 5 - 9.
- b. Addison Wesley Teacher's Manual Second Edition, chapters 5 - 9.
- c. Activities 1 - 14 Multiplication Division activity section.
- d. Mathematical Skill Builder records.
- e. Scott Foresman Teacher's Manual (paperback) activities 21 -36.
- f. Cyclo Reader disks M22 - M44.
- g Pupil Authored problems.
- h. Duplicator Masters for Grade 3.
- i. Continental Press Duplicating Masters Grade 3. Parts 2 and 3.



**MULTIPLICATION AND DIVISION  
MASTERY OF FACTS - COMPUTATIONAL SKILLS**

III REAL NUMBER  
OPERATIONS.  
Multiplication  
and Division  
3. S  
4. S  
R. 5, 6  
E. 5, 6

6. Given the 100 multiplication fact test, the learner can write the answers with 95% accuracy in 5 minutes

7. Given the 100 division facts test, the learner can write the answers with 90% accuracy in 7 minutes.

8. Given a group of problems such as:

$$579 \times 3 \times 6 = \begin{array}{r} 6379 \\ \underline{\quad 3 \quad} \end{array}$$

$$\begin{array}{r} 785 \\ \underline{75} \end{array} \quad \begin{array}{r} 9399 \\ \underline{\quad 4 \quad} \end{array} \quad \begin{array}{r} 607 \\ \underline{791} \end{array}$$

$$\begin{array}{r} 394 \\ \underline{485} \end{array} \quad \begin{array}{r} 725 \\ \underline{903} \end{array}$$

The learner can compute them with 85% accuracy.

9. Given a group of problems such as:

$$2) \overline{672} \quad 8) \overline{7534}$$

$$31) \overline{206} \quad 78) \overline{643}$$

$$837 \div 31 =$$

He can compute them with 85% accuracy.

6, 7, 8, 9

a. Addison Wesley Text 3 -units 5 - 9.

b. Addison Wesley Text 4. Units 4 - 8.

c. All activities listed in Addison Wesley Teacher's Manual, second edition; Text 3, Units 4, 5, 6, 8, and 9. Text 4, units 5 - 9.

d. All activities listed in Multipli-  
cation-Division activities section.

e. Cyclo Teacher Disks M22 - M44.

f. Old Scott Foresman paperback Teacher Manual Activities 13 - 36; Pages 337 - 351.

g. Student authored problems.

h. Continental Press Duplicating Masters Grade 3, Parts 2 and 3; grade 4 Parts 1, 2, and 3.

i. Addison Wesley Duplicator Masters Grades 3, 4, 5, and 6.

**SPECIAL PROPERTIES OF 0 IN  
MULTIPLICATION AND DIVISION.**

**III REAL NUMBER  
OPERATIONS  
Multiplication and  
Division**

4. S  
5. R.  
6. R

10. Given a set of 20 problems in multiplication and division involving zero, the learner can write the answers with 85% accuracy.

$8 \times 0 =$ ,  $21 \times 0 =$ ,  $0 \times 15 =$ ,

$20 \div 0 =$ ,  $14 \div 0 =$

10. a. Addison Wesley Text Grade 4.  
Pages 102-103; 114 - 115.

b. Valuable article The Arithmetic Teacher, May 1969, Zero, the Trouble Maker. P. 365 - 367.

c. Emphasize the fact that zero should be twins as it has to perform dual roles. In 93,104, zero represents a number, while in 93,000 zero acts as a place holder.

d. Emphasize the fact that the role of 0 in division is undefined.

e. Student authored problems.

f. Addison Wesley workbook--Grade 4, pages 28 and 33.

g. Game--Picking up zeros.

Have the student pretend that he is picking up zeros from the floor and placing them on his desk. This helps to show him that zero represents an empty set, therefore could not be used as a divisor. When used as a multiplier the product would always be zero.

MORE DIFFICULT MULTIPLICATION AND  
DIVISION FACTS AND PROBLEM SOLVING

III. REAL NUMBER  
OPERATIONS  
Multiplication  
and Division.

4. I  
5. S  
6. S

11. Given problems such as :

$$\begin{array}{r} 4359 \\ 643 \end{array} \quad 48) 1964$$

plus several story problems using these operations, the learner should be able to work them with 85% accuracy, using a check system to verify his accuracy.

11. a. Addison Wesley Text 5. Units 2 - 5.

b. Addison Wesley Text 6. Unit 3

c. All activities in Addison Wesley Teacher's Manuals, second Edition texts 5 and 6. Grade 5 Unit 2 - 6. Grade 6 Units 1 -3.

d. All activities in Multiplication, Division Activity section.

e. Cyclo Reader Wheel H. 43.

f. Student Authored problems.

g. Continental Press Duplicating masters Grade 5, part 1 and 2. Grade 6 part 1 and 2.

h. Addison Wesley Duplicator Masters Grades 4, 5, and 6.

ESTIMATION IN DIVISION.

III. REAL NUMBER  
OPERATIONS.  
Multiplication  
and Division.

- 4. S
- 5. R, E
- 6. R, E

12. Given a test of long division problems, the learner can use estimation in solving these problems.

Evaluation--

Teacher made tests--Sources  
Keeping in Touch pages and  
chapter review, pages 168,  
159 Grade 4. Addison  
Wesley text.

12. a. Addison Wesley text Grade 4,  
Unit 8.

b. Addison Wesley Teacher's Manual,  
second edition, p. 163.

- c. Container estimation.

Obtain an assortment of containers of  
all shapes, sizes and kinds. Such as  
strawberry or fruit boxes, mayonnaise  
jars, small pails, cottage cheese con-  
tainers, oatmeal boxes, cold cream  
jars.

Have on hand rice, lima beans, 1-inch  
cubes, and many other small objects.

Examples of estimation.

Number of beans in a jar.

Fill square and round containers of  
about the same size--fill with beans or  
rice. See which container holds  
more.

Prove that two equal measurements are  
equal by using two different measuring  
instruments--a quart bottle and two  
pint bottles.

Teach children that estimation is  
a useful tool by using it during the  
year. Have children estimate which  
child is the tallest, shortest,  
heaviest, lightest, etc.

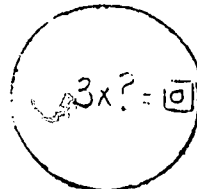
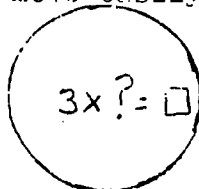
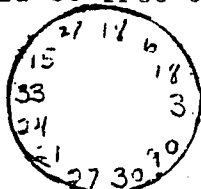
CAUTION-- If a class member is obese,  
avoid estimating children's weights.

See activity sheets for added Estima-  
tion helps.

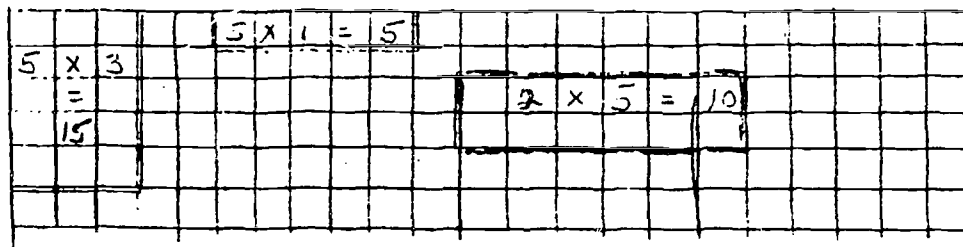
# MULTIPLICATION AND DIVISION ACTIVITIES KEYED TO INSTRUCTIONAL OBJECTIVES

## 1. Multiplication Wheel Instructional Objectives - 1-12

Cut two circles about 7" across out of construction paper. On one circle write several products for number facts around the edge. On the other circle cut a square so the numbers on the first circle will show through. Next to the square write: Example  $3 \times ? =$ . Fasten the circles together in the center with a paper fastener. The circles should be free enough to move easily.



2. Graph Paper Multiplication--On a piece of graph paper have each child draw a picture of a multiplication fact by tracing around the number of rows and columns indicated by certain multiplication problems. Show them how different problems can be equal like  $5 \times 3 = (5 \times 2) + (5 \times 1)$



## 3. Race Track - 1-12

Draw an oval race track on a large piece of paper. Divide it into 12 or 14 spaces. Divide the class into 2 teams, assigning each a cutout horse as a marker. Alternating from team to team, holdup cards containing problems to be worked without pencil and paper. The cards might contain problems such as  $3 \times 2 =$  for the beginners to  $9 \times 69$  for the advanced students. The team advances one space for each correct answer.

## 4. Matching Game -1-12

Print multiplication and division facts, one to a card. Put answers on other cards. Each player draws 4 cards--remaining cards are placed in the middle of the table in a "bonepile". The player on the left of the dealer starts the game by giving the fact of answer appearing on one of his cards, naming another player to supply the answer of fact. The player thus named must surrender that card to the player if he has it. If not, the player draws a card from the "bone pile". If he is successful in either event, he lays the "pair" down on the table. Play now passes to the left, whether or not the caller has been successful in obtaining a set.

5. The Tables Train - 1-12

Children make a train, each member being one car, except the first one who fills the position of engineer. Starting a multiplication Fact table, proceed from the engineer to the box cars to the caboose at the end of the train. If a "fact" is missed, the next car has the opportunity to give the correct answer. If able to do so, he advances to the engineer's seat. Each child moves back one to make room for the engineer.

6. Lotto Multiplication Drill - 2-12

Make boards divided into 25 squares. In each square put the answer of any multiplication fact. Counters are provided for each player. Another student calls out a multiplication fact. Players place counters on the correct answers they find on their cards. Winning cards can be filled vertically, horizontally or diagonally.

7. Over the River - 2-12

Two teams, one on each side of the river. Show a flash card. The first members of each team will compete. The first player to answer remains on his side of the river, but the loser must cross over the river and join the other team, going to the end of the opposite team's line. The longest line wins the game.

8. Boiling Pot. - 2-12

Mark off an area to be called the Boiling Pot. Hold up a multiplication or division fact card. Call two children's names. The last to give the correct answer just sit in the "Boiling Pot." Now hold up a new card, calling one child's name. The person in the "Boiling Pot" competes with the child whose name has just been called. If he is able to answer correctly before the other child does, he may leave the "Boiling Pot" and the other child must go to the pot. If a child does not succeed in getting out of the pot after the third try, appoint another child to take his place.

9. Traveling - 2-12

First child in the row stands beside the child in seat behind him. Seated child holds up a flash card. If standing child answers correctly, he moves to the next child in the row. He may move backward seat by seat until he misses an answer. A scorekeeper is needed to keep score of correct responses.

10. Who's the Winner? - 2-12

Write as many problems on the board as there are players. Divide the group into two teams. When I say "Go," the first player on each team will run to the board and work any problem that he selects and takes the chalk back to the next person in the line. Watch the other team's work. If an error is made and you can call attention to it, the next player on that team must correct it before he can work his problem. If the mistake is not noticed by the opposite team, the problem need not be corrected.

11. Baseball - 2-12

Divide class into two teams. Draw diamond on blackboard. A baseman guards first base. The pitcher calls two different problems, one for each player to solve. At a given signal, they begin to solve their respective problems. At the end of one minute, the umpire calls time and they proceed on to second base, then to third and on home, having worked a problem at each base. If all four problems are worked correctly he wins a point for his team. (Home Run.) This game can be used for many different different processes in math. If only one player works problem correctly, he can proceed while the other player just remain on base until he can answer a problem correctly.

12. Newspaper Problems. 2-12

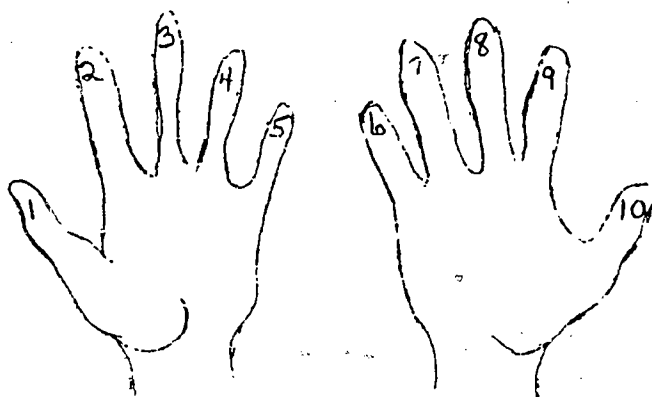
Clippings taken from newspapers pertaining to multiplication, division, percentages etc. A clipping is chosen, posted on note paper. Then an original problem is written for it and solved. As an added exercise an additional problem of similar difficulty should be chosen from the text book and worked.

13. Multiplication Relay. - 2-12

Divide the class into three teams. Line teams up facing chalkboard. Give each team a number i. e. 6, 7, 8. At the signal the first member of each team races to the board and writes the answer to any fact in his assigned table, returns to the line and hands chalk to the next who proceeds to the board and writes the answer to another fact. The facts need not be written in order. The winner scores the last multiple written by his team plus the last multiple written by the other two teams.

14. Hand Multiplication - 2-12

Number the fingers on your hands 1-10, from the left thumb to the right thumb. Place your hands on the desk with palms up, or in front of the player with palms facing the player. To show  $4 \times 9$ , bend down the finger that is numbered 4. The finger to the left of this bent finger represent the number of tens of this product. The fingers to the right of this bent finger represent the number of ones in the product.



15. Catcher's Mitt - 3-12

Catcher's Mitt cut from tagboard with a two inch hole in the center fastened to the end of a ruler or a stick is the tool used by the player. Throw the player a problem on a flashcard or call out a problem to him. He will catch it by moving his catcher's mitt along a hundreds chart until he finds the correct answer and indicates his answer by making it appear in the hole of his mitt. Each "catch" will score one point. He may continue to catch until he makes an error. One child will be given the responsibility of being score keeper.

16. Spinning Game. - 3-12

Required: A hundred number square and three dice or spinners (these can be home-made) with varying instructions written on them instead of the usual spots.

Directions: Two or more players each start at No. 1 on the board and shake the 3 dice (or spin the 3 spinners). A choice of only one of the three results which turn up is made, e. g. from

double		add 9		treble
on	or	on the	or	on the
one dice		other dice		last dice

one instruction can be chosen. Each player does this in turn. The winner is the first player to reach a chosen number, e.g. by changing the instructions on the dice, and it can be extended as the children become familiar with more relationships.

17. Distributive Property of Multiplication over Addition - 3-12

1. This man has 14 buttons his overcoat; 7 on the left, 7 on the right.  $2 \times 7 = 14$ .

2. He now puts his belt on. Above belt 4 buttons on the left and 4 buttons on the right.  $2 \times 4$

Below belt, 3 buttons on the right and 3 buttons on the left:  
 $2 \times 3$ .

Altogether  $4 + 3$  on the left and  $4 + 3$  on the right.  
 $2 \times 4 + 3 = 2 \times 4 + 2 \times 3$ . This can be written as  
 $2(4 + 3) = 2 \times 4 + 2 \times 3$ .

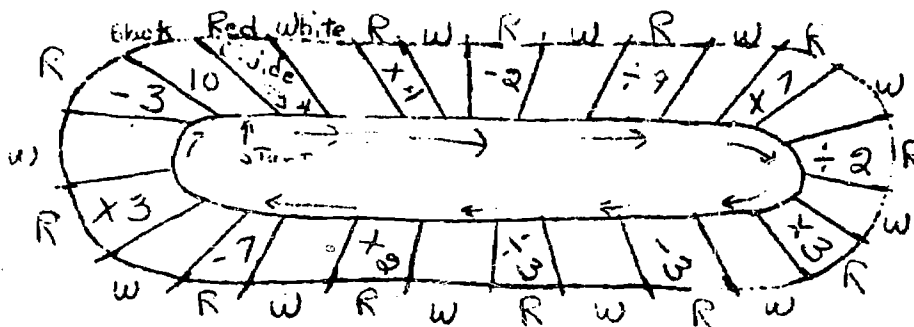
Putting the belt farther down, you would have  $2 \times 5 + 2 =$   
 $2 \times 5 + 2 = 2 \times 5 + 2 \times 2$  or  $2(5 + 2) + (2 \times 2)$





18. Multiplication and Division Practice. - 2-12

Start at 10 and move in direction of the arrows. Write answers in the blank space. What is your answer?



19. Number 9 Family Charts. - 3 -12

$$\begin{aligned} 9 \times 1 &= 9 \\ 9 \times 2 &= 18 \\ 9 \times 3 &= 27 \\ 9 \times 4 &= 36 \\ 9 \times 5 &= 45 \\ 9 \times 6 &= 54 \\ 9 \times 7 &= 63 \\ 9 \times 8 &= 72 \\ 9 \times 9 &= 81 \end{aligned}$$

$$\begin{array}{r} 12,345,679 \\ \times 9 \\ \hline 111,111,111 \end{array}$$

$$\begin{array}{r} 12,345,679 \\ \times 9 \\ \hline 222,222,222 \end{array}$$

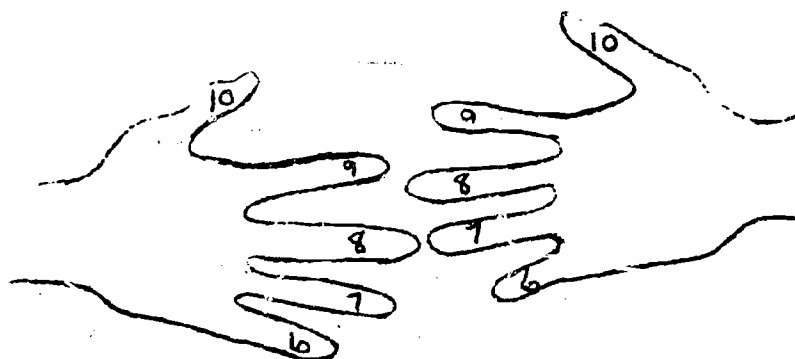
$$\begin{array}{r} 12,345,679 \\ \times 9 \\ \hline 333,333,333 \end{array}$$

19 A Multiplication Wheel - 3-12

Laidlaw, Grade r, Teacher's Manual p. 71

20. Finger Multiplication. - 3-12

Number the fingers of each hand from 6 - 10, starting with the little finger. To multiply  $8 \times 7$ , let the fingers which represent  $8 \times 7$  touch. Take the number of fingers below the touching fingers (3) and add the two fingers that touch (5). This is the number of tens in the product of  $5 \times 10$ . Multiply the number of fingers on each hand above the touching fingers ( $2 \times 3 = 6$ ). So  $8 \times 7 = 50 + 6 = 56$ .



## 21. Football - 3-12

Draw 11 parallel lines on the board, marking them with numerals from 0 - 90, representing the ten-yard lines on a football field. At the other end of each line put a numeral suitable for multiplication facts. Make five small tagboard footballs with a digit from 0 to 9 on each. Divide the players into two teams. Team A will carry the ball first. The first player draws a football from the box, multiplies the number on his football by the number on the 10 yard line in 20 seconds. Make standards stiff--making touchdowns on the field is not a snap. He can keep on playing as long as he can answer correctly in the specified time, advancing from the ten to the twenty yard line and so on until he misses. The ball then goes to the other side when he fumbles. This can be used for  $+$ ,  $-$ , and  $\div$  as well.

## 22. Divisibility - 3-12

To decide whether or not a number is divisible by 3 without a remainder, look at the figures in the numeral. Find the sum. Can the answer be equally divided by 3? If so the numeral can be divided by 3.

By 4. Look at the last two figures in the numeral. If the number expressed by them can be divided by 4, the number itself can also be divided by 4.

By 5. Any number whose numeral ends in 0 or 5 can be divided by 5.

By 6. A number is divisible by 6 if it can be divided by both 2 and 3.

By 7. Is 488 divisible by 7?

$$\begin{array}{r} 488 \\ - 161 \\ \hline 32 \end{array}$$

Since 32 is not divisible by 7, 488 is not divisible by 7.

Is 266 divisible by 7?

$$\begin{array}{r} 266 \\ - 121 \\ \hline 14 \end{array}$$

Since 14 is divisible by 7, 266 is divisible by 7.

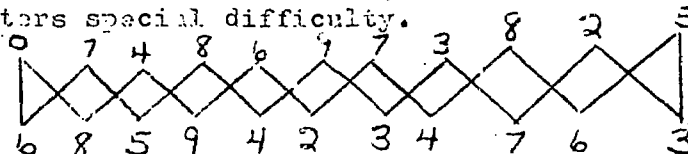
What did we do? The digit in the ones place is isolated from the number and multiplied by 2. Then this number is subtracted from the remainder, beginning with the tens place. If the remainder is divisible by 7 the original number is divisible by 7, and conversely.

By 9

A number is divisible by 9 if the sum of its digits can be divided by 9.

### 23. Crisscross Multiplication - 3-12

Make a sketch on the chalk board, using the numbers that have been giving the youngsters special difficulty.



The player begins with either of the left-hand numbers. Follow the diagonal lines, multiplying the number by the one that follows. Each product correctly given earns one point for the player. Children can make their own crisscross games for the facts which they have difficulty mastering.

### 24. Estimation. - 3-12

A group of child en sit in a circle. At a signal all the children extend any or all of their fingers. Each child then makes an immediate estimate of the number of fingers shown. All fingers are then added to see who is nearest to the exact number shown.

Other members of the class could write their estimates. Other objects could be substituted for fingers.

### 25 Rounding Off - 3-12

Line up the class in two teams. The teacher then gives the instruction-- "round these numbers to the nearest 100, ( or ten or thousand.) She then dictates a number such as 12,426. The first two opposing students call off their answers. The first one calling out the right number goes to the end of the line to await another turn. The loser goes to his seat where he attempts to beat the others by writing the correct answer.

### 26; Checks for Multiplication and Division. - 3-12

$$\begin{array}{r} 93 = 1 + 2 = 3 \\ \times \quad 32 \\ \hline 186 \\ 2790 \\ \hline 2976 \end{array} \quad \begin{array}{l} = 5 \\ 15=6 \\ \\ =24=6 \end{array}$$

$$\begin{array}{r} 8 \times 6 = 48 + 4 = 52 = 7 \\ 42 \overline{) 340} \\ \underline{336} \\ 4 \end{array} = 7$$

Multiplication--see above example. Add digits in 93-- $9 + 3 = 12$ . Keep adding until you get a one digit number--  $1 + 2 = 3$ . Do the same in 32. The result will be 5. Multiply  $5 \times 3 = 15 = 6$ .

Add the digits in 2976 and get 24. Add again  $2 + 4 = 6$ . Since the check number in either direction is 6, the example checks.

Division. Check the numbers as for multiplication; multiply the check number of the divisor by the check number of the quotient and add the check number of the remainder. This should give the check number of the dividend if the division is correct.

## 27. Napier's Rods - 3-12

Make a Napier chart like the illustration. Use a red pen to put the numbers in the divided squares, black for the others.

Cut along the vertical lines so that each column is a separate card or rod. You should have 11 rods.

1. Suppose you want to multiply  $369 \times 6$  using Napier's rods.

A. Pick out the 3, 6, and 9 rods and place them side by side. Then place the index rod to the right of these three rods as shown in the second chart.

Look down the index rod to the number 6. To find the product, you must add the numbers named diagonally as shown in the chart. Start at the right and re-name as you add.

2. Suppose you wish to multiply  $482 \times 35$ .

Pick out the 4, 8, and 2 rods and place them side by side. Then place the index rod at the right of these three rods. What is the product of  $482 \times 5$ ? This is the first of your partial products. What is the product of  $482 \times 30$ ? Now add these to partial products.

Check your work by multiplying in the usual way.

0	1	2	3	4	5	6	7	8	9	
0	1	2	3	4	5	6	7	8	9	10
1	0	2	3	4	5	6	7	8	9	10
2	0	1	2	3	4	5	6	7	8	9
3	0	2	1	0	1	2	3	4	5	6
4	0	3	2	1	0	1	2	3	4	5
5	0	4	3	2	1	0	1	2	3	4
6	0	5	4	3	2	1	0	1	2	3
7	0	6	5	4	3	2	1	0	1	2
8	0	7	6	5	4	3	2	1	0	1
9	0	8	7	6	5	4	3	2	1	0

3	6	9	
3	6	9	1
6	12	18	2
9	18	27	3
12	24	36	4
15	30	45	5
18	36	54	6
21	42	63	7

4	8	2	
4	8	2	1
8	16	4	2
12	24	6	3
16	32	8	4
20	40	10	5
24	48	12	6

28. Up The Ladder - 3-12

0			0	20
7		53	1	12
3		46	7	28
1		37	5	12
2		29	3	16
5		17	8	32
4		25	4	36
8		32	6	20
9		22	7	40
6		14	2	24
+3		-8	x7	÷4

29. Muffin Tin Toss - 3-12

Number cups in muffin pan according to the type of problem you wish to stress, addition, multiplication or fractions. Take turns tossing a soft eraser into the muffin cups from a chalk line about three feet from the pan. The number of the cup tells you your score--now you just add or multiply them correctly.

30. Russian Peasant Multiplication - 3-12

Follow the example:

24-x-33

Halve each number on the left and double each number on the right. Always discarding any halves remainin .

13 x 66

--x-132

Cross out any lines beginning with an even number.

3 x 204

add the numbers on the right to obtain the correct answers.

11 x 528  
850

Check by usual method of multiplication. Why does this work?

### 31. Finger Multiplication. - 3-12

Learn the multiplication facts up to  $5 \times 5$ . The numbers through 5 are called the first cycle, 6 through 10 the second cycle, 11 through 15 the third and so on.

To multiply numbers in the second cycle, call the thumb 6, the first finger 7, and so on to the little finger for 10. Do this for both hands.

What is  $7 \times 8$ ? On one hand bring the first finger, the 7 together with the thumb. We shall call these fingers, representing 7, the "tight fingers" in contrast with the three remaining fingers: loose fingers." On the other hand put together the second finger, the 8, together with the thumb and first finger, is "tight fingers" representing 8.

Bring the two groups of "tight fingers" together. How many do you have? FIVE. This number, 5, is the number of tens in your answer. How many loose fingers have you? Two on one hand, three on the other; but this time instead of adding, multiply the numbers. You get 6, and this is the number of ones in the answer. Five tens, six ones 56.

Try  $8 \times 9$ . Three tight fingers plus four tight fingers gives you seven fingers touching. Hence the product includes seven tens. Two loose fingers times one loose finger gives you 2, the number of ones in the product.  $9 \times 8 = 72$ .

Will this always work? Now try  $6 \times 7$ . You will find that (1 + 2) tight fingers indicates 30, and (4 x 3) loose fingers indicates 12; and of course  $30 + 12 = 42$ .

The Third and Fourth Cycles. What happens in the third cycle?

Name the fingers 11 through 15. What is  $13 \times 14$ ? Start with a bonus of 100. Multiply the sum of the tight fingers (7) by 10. Then add the product of the tight fingers (not the loose ones this time).

$$100 + 10(3 + 4) + (3 \times 4) = 182 \\ = 13 \times 14$$

In the same way.

$$12 \times 15 = 100 + 10(2 + 5) + (2 \times 5) = 180$$

For the fourth cycle, the bonus is 200. Multiply the sum of the tight fingers by 20, add the product of the loose fingers.

$$18 \times 18 = 200 + 20(3 + 3) + (2 \times 5) = 324$$

$$16 \times 20 = 200 + 20(1 + 5) + (4 \times 10) = 320$$

## Finger Multiplication Cont'd.

Table for the first ten cycles.

Cycle	Bonus	+	S*	+	Pl
2nd. (6-10)	0		10		L
3rd. (11-15)	100		10		T
4th. (16-20)	200		20		L
5th. (21-25)	400		20		T
6th. (26-30)	600		30		L
7th. (31-35)	900		30		T
8th. (36-40)	1,200		40		L
9th. (41-45)	1,600		50		T
10th. (46-50)	2,000		50		L

\*Sum of the tight fingers multiplied by the number shown in the column.

! Product of the loose fingers (L) or the (T) fingers.

### 32. Effortless Multiplication. - 3-12

Effortless multiplication is a method of diagonal placement of subproducts to eliminate the need for mental "carrying."

#### Step 1

7	3	7863	7863	7863	7363	7863	7863	7863
<u>94</u>	<u>94</u>	<u>94</u>	<u>94</u>	<u>94</u>	<u>94</u>	<u>94</u>	<u>94</u>	<u>94</u>
12	212	3212	23212	23212	23212	23212	23212	23212
	4	24	824	824	824	824	824	824
				27	527	7527	67527	
					4	24	324	

The first sub-product, 12 (4x3), is placed horizontally as usual. The next sub-product, 24 (4x6), is placed diagonally to the left of the first sub-product, 12. Likewise the third sub-product is placed diagonally to the left of the preceding one until you have completed the multiplication of all digits by the first digit in the multiplier.

In starting the process with the digit in the "tens" place the answer horizontally in the third line. In this illustration (27). The process is repeated following the pattern for the digit multiplier in "ones" place.

### 33. Multiplication Enrichment--Find the missing digit. #3-12

Each letter stands for a digit. Each time it is used in an example, it stands for the same digit. The letter stands for 0.

$$\begin{array}{r} \text{A} \text{ L} \\ 2 \text{ F} \\ \hline \text{A} \text{ 4 } 3 \\ 5 \text{ R } 0 \end{array}$$

Look at the code names for the partial products. What is the sum of 3 and 0? Then what number does B stand for in the code name for the product?

What added to 4 equals 8? Is the second partial product equal to 540?

You know that the second partial product is 540 and is the result of multiplying the first factor by 20. What number multiplied by 20 equals 540? Then what is the factor that is represented by A L?

Since A stands for 2, the first partial product is 243. The first factor is 27. What number times 27 equals 243? Then what number does F stand for? Write the multiplication complete with all the digits that are now represented in code in the example above.

$$\begin{array}{r} \text{x} \quad 2 \text{ 7} \\ \quad 2 \text{ 9} \\ \hline 2 \text{ 4 } 3 \\ 5 \text{ 4 } 0 \\ \hline 7 \text{ 8 } 3 \end{array}$$

Here are some other examples to play with for a few minutes.

$$\begin{array}{r} \text{X} \quad \text{Q } 1 \text{ 3} \\ \quad \text{4 } 6 \\ \hline \text{L, Q } \text{W } \text{S} \\ \text{S, 5 } \text{Q } \text{O} \\ \hline \text{T, W } \text{T } \text{S} \end{array}$$

$$\begin{array}{r} \text{E } \text{E } \text{O} \\ \quad 7 \text{ 8} \\ \hline \text{B } \text{B } \text{O} \\ \text{A, A } \text{O } \text{O} \\ \hline \text{B, L } \text{B } \text{O} \end{array}$$

$$\begin{array}{r} \text{B } \text{O } \text{Y} \\ \quad 2 \text{ 5} \\ \hline \text{A, R } \text{B } \text{O} \\ \text{6, 1 } 2 \text{ O} \\ \hline \text{G, Y } \text{R } \text{O} \end{array}$$

$$\begin{array}{r} \text{A } \text{O } \text{B} \\ \quad \text{C } \text{D} \\ \hline \text{D } \text{A } \text{D} \\ \text{C, 2 } 4 \text{ O} \\ \hline \text{C, } \text{D } \text{D} \end{array}$$

$$\begin{array}{r} \text{C } \text{E } \text{D} \\ \quad \text{G } \text{J} \\ \hline \text{D } \text{D } \text{I} \\ 2, 5 \text{ 7 } \text{O} \\ \hline \text{J, J } 4 \text{ O} \end{array}$$

$$\begin{array}{r} \text{aQ } \text{D } \text{N} \\ \quad \text{C } \text{N} \\ \hline \text{A, O } \text{E } \text{N} \\ \text{D C, A } \text{O } \text{O} \\ \hline 1 \text{ 5 } 3 \text{ 7 } 5 \end{array}$$

$$\begin{array}{r} \text{A } \text{T} \\ \quad 9 \text{ R} \\ \hline \text{A } \text{O } \text{R} \\ 2, \text{ N } \text{F } \text{O} \\ \hline \text{A, K } \text{F } \text{R} \end{array}$$

$$\begin{array}{r} 9 \text{ T} \\ \quad \text{P } \text{S} \\ \hline 1 \text{ 9 } \text{P} \\ \text{Q, T } \text{T } \text{O} \\ \hline \text{P, O } \text{T } 6 \end{array}$$



# Division Enrichment - 3-12

Can you break the code? Remember that 0 always stands for zero.

What number equals 2 - 0? Then C stands for 2. What is the divisor in this example?

To find the number represented by L L ,0.0.0. find the product of 4,000 x 22.

In the example X minus L stands for one. Since L stands for 0, then what number does X stand for?

Find the other numbers represented by letters. Then write the division example complete with all its digits.

Break the code in all these division examples.

$$\begin{array}{r} 2 \text{ c) } \overline{8 \text{ X, B C } 2} \\ \underline{L \text{ L, C C } 0} \\ 1, \text{ B C C} \\ \underline{\text{B, B } 0 \text{ C}} \\ \text{C } 2 \\ \underline{2 \text{ C}} \\ 0 \end{array}$$

$$\begin{array}{r} \text{P) } \overline{\text{X J J}} \\ \underline{\text{H, C M L}} \\ \text{A, J } 0 \\ \underline{1 \text{ 4 } 1} \\ \text{L H } 0 \\ \underline{\text{H } 1} \\ \text{H L} \end{array}$$

$$\begin{array}{r} \text{R R ) } \overline{9, \text{ A } 5 \text{ R}} \\ \underline{\text{R O A, L C R}} \\ \text{A A, } 0 \text{ 0 } 0 \\ \underline{\text{R O, L C R}} \\ \text{A, A } 0 \text{ 0 } \\ \underline{5 \text{ C R}} \\ \text{2 } 0 \text{ 0 } \\ \underline{1 \text{ R}} \\ \text{R R} \\ 0 \end{array}$$

$$\begin{array}{r} 2 \text{ r) } \overline{4, \text{ Z } 5 \text{ Z}} \\ \underline{\text{J J, P M Y}} \\ \text{J } 0, 0 \text{ 0 } 0 \\ \underline{3, \text{ P M Y}} \\ \text{M, Y } 0 \text{ 0 } \\ \underline{\text{Z, M M Y}} \\ \text{Z, M C } 0 \\ \underline{2 \text{ Y}} \\ \text{M Y} \\ 0 \end{array}$$

$$\begin{array}{r} \text{H } 5) \overline{4, \text{ N O P}} \\ \underline{\text{X N, X } 3 \text{ T}} \\ \text{X } 0, 0 \text{ 0 } 0 \\ \underline{1, \text{ X } 3 \text{ T}} \\ \text{N, T } 0 \text{ 0 } \\ \underline{\text{N } 3 \text{ T}} \\ \text{N R T} \\ 0 \end{array}$$

35. Birthday Cake - 3-12

Write these numbers across

2.  $37 \times 3$

4.  $271 \times 41$

6.  $459 \times 239$

Write these numbers down.

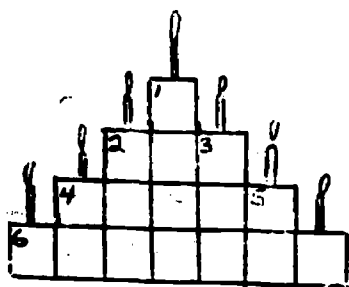
1.  $101 \times 11$

2.  $555 - 5$

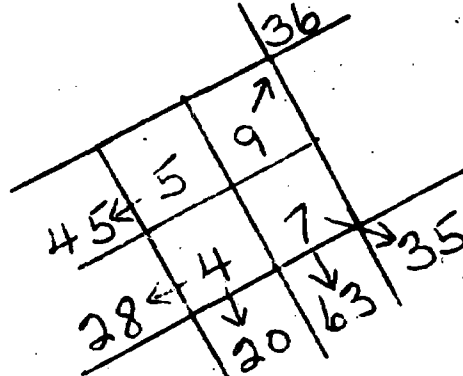
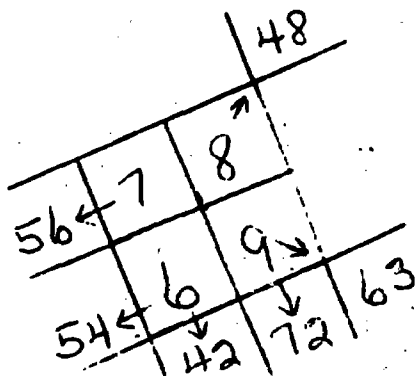
3.  $37 \times 9 \div 3$

4.  $22 - 2$

5.  $11 \times 1$



36. Duplicate a page of grids like the ones below. Have pupils write numerals below 10 in the numbered boxes. Answers to these facts should then be written in the other boxes. Have the children then place arrows to the answers for the different fact problems.



37. Multiplication or addition can be used to complete each box below. (1) read across, (2) read down. The product of the factors in the right column should equal the product of the factors shown across the bottom. More advanced students can use a 4 square grid.

2	4	8	
3	5	15	
6	20	120	

2	3	2	12
2	3	1	6
2	4	5	40
8	36	10	3080

## 3B. Built-In Place Value Chart for Long Division. - 3-12

Whenever children work with quotients larger than 10, the use of lined paper provides a built-in chart for place value. Have the student turn his note paper sideways.

4	8	2	3	7	9	6
		1	9	2	0	0
			4	5	9	6
			4	3	2	0
				2	7	6
				2	4	0

4	0	0
	9	0
		5
4	9	5

R 36

39. Practice in thinking of more than one operation at a time. 2-12

Sum	12	9	15	17		16	38	13
Part	5	4	7		14	7		
Part	7	5	8		10			
Product	35	20		72		63	72	36

40. Lattice Multiplication. ~ 3-12

Handwritten numbers in a 3x3 grid:

1	6	9
6	3	2
9	2	2

Handwritten numbers in a 4x4 grid:

1	2	8	8
2	3	5	0
8	1	1	2
8	0	2	2

Dashed lines connect the numbers in the 3x3 grid to the 4x4 grid.

FRACTION NUMBERS -  $1/2$ ,  $1/4$ , VOCABULARY

III. REAL NUMBER  
OPERATIONS---  
FRACTIONS

K-I

1. Learner intuitively works with vocabulary development and recognition of parts of a whole or parts of a set.

- 1a. Vocabulary words to be developed are:  
part, some, all, whole, half, one fourth.
- b. To develop the idea of equality of fractional parts:
- a. divide and share apples or candy
  - b. fold or cut paper
  - c. divide beads, blocks, clay
  - c. Use of cuisenaire rods if available.
  - d. See activities #1-3.
  - e. Evaluation
- Does the child understand the process of dividing  $1/2$  an apple and sharing with others? (object is to make fourths)

# FRACTION NUMBERS $\frac{1}{2}$ , $\frac{1}{3}$

## III. FRACTIONS

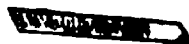
G1-S, G2-R

2. When asked, the learner will be able to differentiate between a set of two and two halves, a set of three and three thirds, etc. with 100% efficiency.

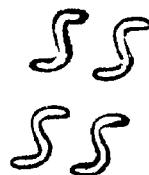
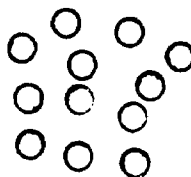
Readiness: The learner has previously been exposed to  $\frac{1}{2}$  and  $\frac{1}{3}$  in kindergarten.

- 2a. See activity 1 in activity section under Fractions.
- b. Putting a mark or color on pictures that show thirds.
- c. See Addison-Wesley teacher's guide, pp. 201-203, Book 1
- d. See activities #2 and 3
- e. See activity #4. Use as much as fits your needs.
- f. Use Teaching Fraction transparencies
- g. Use pp. 78-79 from Addison-Wesley Duplicator Masters, Second Ed. Bk 1
- h. Filmstrip #118, "What is a Fraction?"
- i. Evaluation
- Learner can with teacher supervision, color fractional areas as indicated by the sample below, or write fractional numerals.

Write  $\frac{1}{2}$  under regions that are half colored.



Ring half the objects in each set.  
(Continue this with thirds)



FRACTIONAL NUMBERS AND NUMERALS ASSOCIATED  
WITH THE FRACTIONAL NUMBERS  $\frac{1}{2}$ ,  $\frac{1}{3}$ ,  $\frac{1}{4}$ ,  
 $\frac{2}{3}$ ,  $\frac{3}{4}$

III. FRACTIONS

G2-S, G3-R

3. When asked the learner will be able to identify fractional numbers and numerals associated with  $\frac{1}{4}$ ,  $\frac{2}{3}$ ,  $\frac{3}{4}$ , with 100% (or near) efficiency.

Readiness: Learner is aware of fractional parts and has worked with  $\frac{1}{2}$  and  $\frac{1}{3}$  intuitively.

- 3a. Activities 1-4 can be used for review  
b. See activities 5, 6, 7  
c. See Addison-Wesley, pp. 275-280-BK 2  
d. Use Teaching Fractions transparencies  
e. Use appropriate sheet from duplicator masters, Book 2, Addison-Wesley 2nd edition  
f. Filmstrips: #118, "What is a Fraction"  
g. For evaluation measure, see Merrill Book Co. Discovering Mathematics, p. 58

FRACTIONAL NUMBERS INCLUDING HALVES,  
THIRDS, FOURTHS, SIXTHS, AND EIGHTHS

III. FRACTIONS

G3-S, G4-K

4. The learner will be able to identify and write all fractions through eighths with 100% (or near) efficiency.

Readiness: Previous experience has been gained in grade two with halves, thirds, and fourths.

- 4a. Use preparatory practice listed under "Preparation," pp. 336-340 of Addison-Wesley Teacher's Guide Book 3.
- b. Use pages 336-337 of Addison-Wesley Teacher's Guide or 298-299 of child's text for practice in review.
- c. See activities 1-7 for review.
- d. See activity 8 to further objective one.
- e. Use Addison-Wesley, Book 3, pp. 338-341 of Teacher's Guide to further objective 1.
- f. Use teaching fractions transparencies.
- g. Filmstrips #118, "What is a Fraction?" #119, "How Large Is a Fraction?" #120, "Fractions of a Group"
- h. For evaluation of objectives see Activity 9.

5. Learner will be able to solve five simple word problems related to fractions through eighths with 85% accuracy.

- 5a. Suitable problems may be found in any text for this level.
- b. See activity 10.
- c. Evaluation  
Thought problems p. 343 of Teacher's Guide, Addison-Wesley, Book 3.  
Children's answers need not be in reduced form, however.

GIVEN FRACTION AS A FRACTIONAL  
PART OF THE WHOLE

III. FRACTIONS

G4-S, G5-R

6. Learner can, given 10 problems identifying fractions, recognize and write the fractions with 90% efficiency.

**Readiness:** Learner has had previous experience in fractional recognition.

- 6a. See Addison-Wesley, Book 4, pp. 240-250, for review and mastery
- b. Addison-Wesley, Book 5, pp. 186-191, may be of help
- c. Fraction discs or cut up paper plates
- d. Fraction Wheel, Ideal School Supply, Chicago
- e. Bulletin Board Charts (a kit) containing materials on fractions, common measures, and fundamental processes F.A. Owen Co., Dansville, New Jersey
- f. Use Duplicator Masters, Book 4, Addison-Wesley - appropriate pages
- g. See activity 10
- h. Filmstrips - #119 "How Large is a Fraction?", #120 "Fractions of a Group"
- i. Teaching Fractions transparencies  
Evaluation could be done by a teacher made test such as:
  - ① . . . Write the fraction that
  - ② . . . tells what part of the set is black



EQUIVALENT AND NON-EQUIVALENT FRACTIONS

III. FRACTIONS

G4-I, G5-S, G6-R

7. The learner will be able to define and name equivalent fraction sets with 85% efficiency.

Readiness: Learner by this time should have achieved near mastery in fraction recognition.

- 7a. Addison-Wesley, Book 4, pp. 252-268
- b. Addison-Wesley, Book 5, pp. 194-203
- c. Addison-Wesley workbook, pp. 62-65 provide practice exercises
- d. An expanded classroom chart such as in activity 7 would be helpful.
- e. For evaluation, the above mentioned workbook pages may be useful

8. Learner will be able to determine whether a given fraction is less than, greater than, or equal to another given fraction with 85% efficiency.

- 8a. Addison-Wesley, Book 5, pp. 204-205, 226-227
- b. Addison-Wesley, workbook, pp. 70-71 provide practice
- c. Through the use of fraction discs, individual cut outs, paper plate sections, etc., pupils can compare fractional sets.
- d. See activity 10
- e. Continental Press, Transitional (modern) Math ditto packs -  $\frac{1}{2}$  and  $\frac{1}{4}$
- f. Use Teaching Fractions transparencies
- g. Filmstrips: #335 "Working With Like and Improper Fractions"
- h. Evaluation checks may be found in Addison-Wesley, Book 4, pp. 266-267

NUMERATOR AND DENOMINATOR
---------------------------

III. FRACTIONSG4-I, G5-S, G6-R

9. Learner will be able to define and identify numerator and denominator when asked to do so. Primarily, this should be a convenience in language and discussion of fractions.

Readiness: Learner by this time can understand the fraction concept.

- 9a. Addison-Wesley, Book 4, pp. 258-259 serve as introduction  
b. Addison-Wesley, Book 5, pp. 190-191  
c. Practice exercises can be found in Continental Press (modern) 5<sup>1</sup>  
d. Use appropriate pages in Duplicator Masters, Book 4, 2nd edition, Addison-Wesley  
e. Continued use of terminology at this time should facilitate ease of usage on the part of the learner. This is the evaluation.

# IMPROPER FRACTIONS AND MIXED NUMBERS

## II. FRACTIONS

G4-I, G5-S, G6-R

10. Learner should be able to recognize and explain a fraction containing a numerator as large or larger than the denominator. Proficiency may be gained by repeated usage of such fractions.

Readiness: Learner can identify and define numerator and denominator.

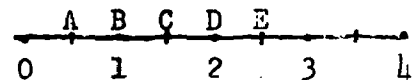
- 10a. Addison-Wesley, Book 4, pp. 262-263 provide introductory material  
 b. Addison-Wesley, Book 5, pp. 192 provides introductory material  
 c. Scott Foresman, Book 5, pp. 148-150  
 d. Aladdin Fraction Line Set, #795, Ideal School Supply Co., Oak Lawn, Illinois  
 e. Fraction discs, paper plates, flannel board, fractional parts, etc.  
 f. Continental Press (Modern) Gr. 5<sup>2</sup>, pp. 1, 2, 3, 11  
 g. Evaluation may have to be through teacher observation and teacher made test, since materials are scarce.

A test might be:



Circles are divided into fourths

The shaded areas equal    fourths



Point    shows  $\frac{3}{2}$  on the number line.

IMPROPER FRACTIONS AND MIXED NUMBERS

III. FRACTIONS

G4-I, G5-S, G6-R

11. Learner can change a mixed number to an improper fraction or the reverse with 85% efficiency.

- 11a. Addison-Wesley, Book 4, pp. 298-302
- b. Addison-Wesley, Book 5, pp. 241-242
- c. Practice exercises Continental Press, Modern Math, 5<sup>2</sup>, pp. 23-24
- d. See Scott Foresman Teacher's Manual, pp. 364 for a game called Fraction Match, which supports this objective.
- e. Filmstrips:
  - #124 "Mixed Numbers"
  - #125 "Using Mixed Numbers"
  - #335 "Working With Like and Improper Fractions"
- f. Inventory and evaluation material can be found in Learning to Compute, Harcourt Brace, 2nd edition, pp. 42--43

LOWEST TERM FRACTIONS

III. FRACTIONS

G4-I, G5-S, F6-R

12. Learner will be able to identify the lowest term fraction in a group of equivalent fractions with 85% accuracy.

Readiness: Learner is familiar with meaning of equivalent, can differentiate between equivalent and non-equivalent fractions.

- 12a. Addison-Wesley, Book 4, pp. 270-274 for introduction  
b. Addison-Wesley, Book 5, pp. 206-212 for introduction  
c. Any fraction line set would be helpful  
d. Scott Foresman, Book 5, pp. 143-146  
e. See activity 12 for supplementary work  
f. Harcourt Brace, Learning to Compute, 2nd edition, p. 40  
g. Continental Press, Modern Math, 5<sup>2</sup>, pp. 17-18  
h. See activities 13 and 14  
i. Filmstrips: #332, "Reducing Fractions"  
j. Evaluation might take the following form: Give groups of equivalent fractions and have child circle those in lowest terms. Give groups of higher term fractions and have child write the lowest term.

ADDING AND SUBTRACTING LIKE  
AND UNLIKE FRACTIONS

III. FRACTIONS

G5-S, G6-R

13. Learner will add and subtract like and unlike fractions with 85% efficiency.

Readiness: Review the inverse relationship between addition and subtraction to apply these operations with fractions.

- 13a. Addison-Wesley, Book 5, pp. 244-251 and 254-257 for developmental processes  
b. Addison-Wesley, Book 6, pp. 118-125 for development.  
c. Continental Press Modern Math, 5<sup>2</sup>, pp. 20-21 for practice  
d. See activity 11  
e. Addison-Wesley workbook 5, pp. 74, 76-79  
f. See activity 14, 15, 18  
g. Filmstrips:  
    #335 "Like and Improper Fractions"  
    #333 "Adding With Fractions"  
h. Evaluation listed at bottom of this page

14. Learner will apply above skills in solving five word problems with 80% efficiency.

- 14a. Addison-Wesley, Book 5, p. 257, for experience in problem solving  
b. Addison-Wesley, Book 6, p. 125 for experience  
c. Let children make up their own problems. Select those best suited to ability level and experiences of group and place on math table with answers on back of problem. Individuals can solve as they choose.  
d. Use fraction number line  
e. Evaluation for both objectives:  
    Continental Press, 5<sup>2</sup>, pp. 22, 27, 28

ADDING AND SUBTRACTING MIXED  
NUMBERS AND REGROUPING

III. FRACTIONS

G5-S, G6-R

15. Learner will be able to add and subtract mixed numerals with like and unlike denominators with 85% efficiency.

Readiness: Learner has already gained skill in adding and subtracting fractions with like and unlike denominators.

16. Learner will be able to add and subtract mixed numerals with like and unlike denominators using regrouping with 85% efficiency.

- 15a. Addison-Wesley, Book 5, pp. 258-261 for information
- b. Learning to Compute, Harcourt Brace, 2nd edition: addition p. 54, subtraction p. 59. These are for practice
  - c. Continental Press, Modern Math, 5<sup>2</sup>, pp. 24-25 for practice.
  - d. Scott Foresman, Book 5, pp. 177-182 Another approach to addition
  - e. See activities 12, 13, 15, 16
  - f. Filmstrip #338
  - g. Evaluation: Addison-Wesley workbook 5, p. 80

- 16a. Addison-Wesley, Book 5, p. 262 for introduction
- b. Addison-Wesley, Book 6, pp. 128, 129, 131 for introduction
  - c. Scott Foresman, Book 5, pp. 190-195 another approach
  - d. Learning to Compute, 2nd edition, p. 60
  - e. Use game, Fraction Quizmo
  - f. Evaluation: Addison-Wesley workbook 5, p. 81

ADDING AND SUBTRACTING MIXED NUMBERS  
AND REGROUPING

III. FRACTIONS

G5-S, G6-R

17. Learner will be able to apply skills learned in problem solving by solving five word problems with 85% efficiency.

- 17a. Addison-Wesley, Book 5, p. 263
- b. Scott Foresman, Book 5, pp. 196-197
- c. Addison-Wesley, Book 6, pp. 135-136
- d. Scott Foresman, Book 6, pp. 98, 100, 101
- e. Children may contribute original problems with answers for math table
- f. For evaluation, any test, teacher made or commercial, incorporating the three objectives would be satisfactory. For example: Addison-Wesley, Teacher's Guide Book 6, p. 143, "Short Stories" would be useful.



MULTIPLYING A WHOLE NUMBER  
AND A UNIT FRACTION

III. FRACTIONS

G5-S, G6-R

18. Learner can multiply a whole number and a unit fraction in preparation for multiplying two or more fractions.

Readiness: Learner is able to apply associative and commutative laws in multiplying.

19. Learner will use the zero principle and one as identity number in multiplying fractional parts.

- 18a. Addison-Wesley, Book 5, pp. 286-289  
b. Addison-Wesley, Book 6, pp. 116-119  
c. Use of any fraction line  
d. Addison-Wesley workbook 5, pp. 88-89  
e. Houghton Mifflin, Book 5, pp. 202-203, 206-207  
f. Continental Press, Modern Math 6<sup>1</sup>  
g. See activity 17

- 19a. Addison-Wesley; Book 5; pp. 290-291  
b. Addison-Wesley; Book 6, pp. 150-151  
c. Filmstrip #339, Grades 1-18 for introductory purposes

MULTIPLYING TWO OR MORE  
FRACTIONAL UNITS

III. FRACTIONS

G5-I, G6-S

20. Learner can multiply a given number of examples involving fractional units with 85% efficiency.

Readiness: Learner has previously gained skill in the multiplication of a whole number and a fraction.

- 20a. Addison-Wesley, Book 5, pp. 292-293, 295  
b. Addison-Wesley; Book 6, pp. 152-155  
c. Harcourt Brace, Learning to Compute 2nd edition, pp. 62-63  
d. Continental Press, Modern Math, Book 6  
e. Addison-Wesley workbook, book 5, 2nd edition, pp. 99-103  
f. Addison-Wesley, Book 5, 2nd edition, Teacher's Guide, pp. 300-301  
Involves problem solving  
g. See activities 19-20  
h. Filmstrip:  
#123 "Multiplying Fractions"  
#337 "Multiplying Fractions"  
#339 "Multiplication and Division"

21. Learner can use the short cut method of multiplication of fractions by dividing numerators and denominators by the same factor.

- 21a. Addison-Wesley, Book 6, p. 157 for introduction  
b. Harcourt Brace, Learning to Compute 2nd edition, pp. 64-66 for practice  
c. Addison-Wesley, Book 6, pp. 158-159  
Apply the distributive principle as another approach to multiplying fractions  
d. For evaluation see activity 20

# THE RECIPROCAL PROCESS IN DIVISION

## III. FRACTIONS

G5-I, G6-S

22. Learner can divide a given number of examples involving division of rational numbers by using reciprocal process with 85% efficiency.

Readiness: Learner is able to multiply rational numbers.

- 22a. Addison-Wesley, Book 6, pp. 156, 167-171 intro.; 172-177, problem solve  
 b. Harcourt Brace, 2nd edition, Learning to Compute, pp. 68-73  
 c. Scott Foresman, Book 6, pp. 198-200  
 d. See Grade 6, Addison-Wesley work-book  
 e. Continental Press, Modern Math, Grade 6  
 f. See "Fruit Punch", activity 20 for a culminating experience.  
 g. Filmstrip #339 "Beginning of Multiplying and Dividing Fractions" Frames 19-32  
 #336 Use this after reciprocals  
 h. For evaluation of all aspects of fractions:

add  $1/2$

$1/2$	
$1/2$	
$3/4$	
$5/8$	
$2/3$	

subt.  $2/3$

1	
$1 \frac{1}{3}$	
$1 \frac{1}{6}$	
$5/6$	
?	$1 \frac{1}{2}$

divide by  $3/4$

12	
3	
?	12
$5/8$	

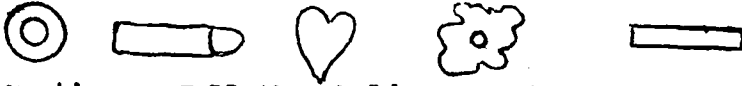
mult. by  $2/3$

6	
12	
9	
$2 \frac{1}{4}$	

## ACTIVITIES - FRACTIONS

### 1. Pieces of Halves or Both k-1

Materials: Make two each of the following objects from paper. Be sure they are large enough to handle conveniently.



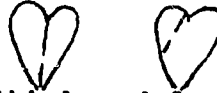
Directions: Tell the children stories. For example:

(1) "You and your friend are hungry and you have only one piece of candy to share between you. What would you do?" The child should say, "I would divide it into halves." Then let him cut it with scissors.

(2) "A friend asks for piece of your doughnut just to see what it tastes like." Child cuts it. "Is this cut in two pieces? In half?"

(3) "Fold the paper heart in two. Are both sides alike? Cut in half."

(4) Have the children draw pictures of objects and show them divided in half or in two pieces with a line.



(5) The same ideas could be used for thirds and fourths.

(6) As the need arises, directions such as "Break this piece of candy in half," or "Fill this glass about half full," can be used to make fractions more meaningful.

### 2. Give each child a set of 10 small counting objects. Suggested directions and questions are:

(1) Put 4 counters on your desk. Divide the set of counters into two sets with the same number in each set. How many counters are in each set?

(2) What is one half of 4?

(3) Continue with 6, 8, and 10 counters.

(4) Follow up: Have children draw a set of 4, a set of 6, of 8 and of 10 objects of their own choosing. They may draw buttons, wheels, tops, hats, etc. Then have them take turns drawing rings around one half of each set. Next, have them write below each picture of a set the number that tells the number of objects in one half the set.

### 3. Let some children cut pictures from magazines that show groups of objects which can be separated into two equal subsets or halves. These pictures may be pasted on a large piece of tagboard (or put on bulletin board with a caption such as "We Can Make Halves." Other children may cut pictures of sets of objects which cannot be separated into halves. These could be posted under a caption "We Cannot Make Halves."

#### 4. How Much Pie? 1-2

**Materials:** Three large paper plates that have been cut into halves, thirds, and fourths; and three circles of construction paper for each child the same size as the paper plates.

**Directions:** The teacher is to request something like the following:

- (1) "Will you take  $\frac{1}{3}$  or a pie next door to Mrs. Smith."
- (2) "You may take  $\frac{1}{4}$  of a pie for your lunch."
- (3) "Please put  $\frac{3}{4}$  of a pie in the picnic basket, Sue." In each case, the child should take the correct portion and pretend to do as directed.

Show how the bottom number means the number of pieces into which the pie was cut, and the top number shows how many pieces were taken, or are left.

Children should understand that pie-cutting is an estimation because no measuring device is used, while a measuring pitcher is more nearly exact.

5. Place four chairs together. Ask a child to move three fourths of the chairs away. Draw a rectangle on the chalk board and have another child color two fourths of the drawing. Repeat with the other fractional parts in related situations.

6. Use these diagrams for teaching fourths, quarters. The clock face can also be worked in at this time.

Color as directed.

white	blue
gray	black

$\frac{1}{4}$  of the drawing is blue

What part of the drawing is black?         

What part of the drawing is gray?         

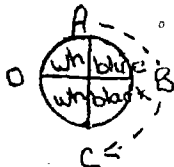
What part of the drawing is white?         



One Fourth of the drawing is blue.

It is also called one quarter.

Quarter is another name for fourth.



Look at the blue and black parts. Together they are what part of the drawing?          Trace the arrows from A to C. How far did you move around the drawing?         

7. Let the children make a classroom fraction chart.

one whole			
one half		one half	
one third	one third	one third	
one fourth	one fourth	one fourth	one fourth

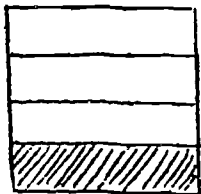
8. Name the Fraction

Materials: Flannel board, fractional cutouts of all sizes (or paper pie plates different colors and cut into fraction parts may be used.)

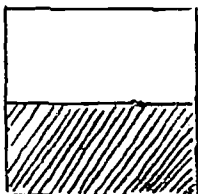
Directions: One child is "It". He puts a fractional part on the flannel board and calls on someone to tell what the fraction is. If the child gets it right, he is "It" and gets to put up a fraction. If necessary, show all the equal parts of the whole.

9. Write the fraction for (a) the shaded region (b) the unshaded region.

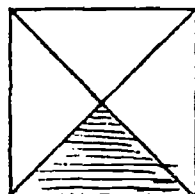
1.



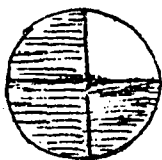
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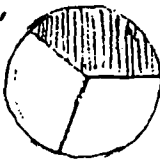
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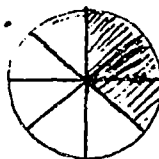
4.



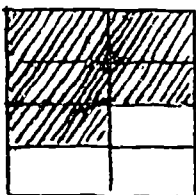
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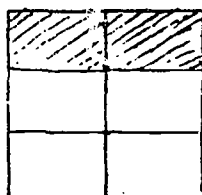
6.



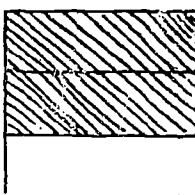
7.



8.



9.



10. Let each pupil make up one or more questions similar to the following:

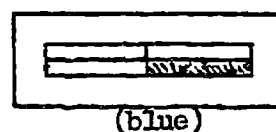
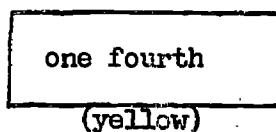
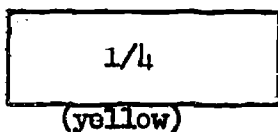
(a) Would you rather have  $\frac{1}{5}$  of a set of ten dollar bills or  $\frac{1}{2}$  of a set of ten dollar bills?

(b) Which is longer, a quarter hour or half hour?

Select the problems that are the most interesting and within the children's ability level. Place them (with answers on reverse side) on the Mathematics Table for individual work.

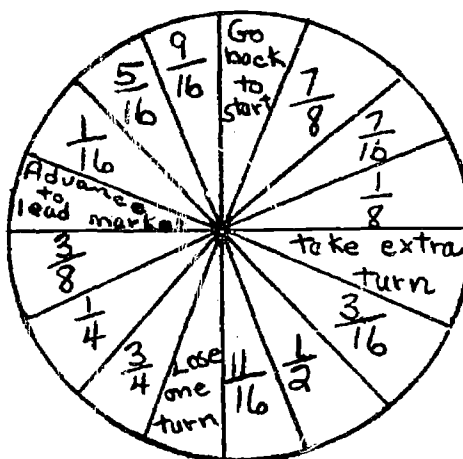
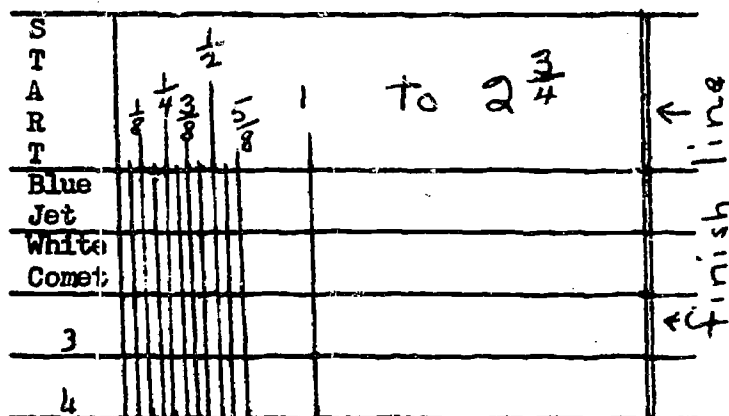
## 11. Over Yellow

Make sets of cards as shown below. Pupils shuffle and deal all the blue cards, then all the yellow cards. A player starts by saying "Over Yellow" which is the signal for each pupil to pass one yellow card to the player on his right if he has one. The pupils try to make as many fraction books as possible containing two yellow and one blue card representing the same fraction. Each pupil takes turns giving the signal until a player goes out. One point is given for each correct book and two points for going out. The pupil with the most points wins.



## 12. Raceway

Prepare a playing-board and a spinner similar to the ones shown below. Four players or teams may compete on each board and each should have a marker, such as the cutout of a racer.



Blue Comet						
Spin	1	2	3	4	5	6
	0	7/16	lost			
	+7/16	+1/4	turn			
	<u>7/16</u>	<u>15/16</u>				

Directions: At the start all markers should be placed at the starting line, and players should take turns twirling the spinner to determine how far to advance the racers each time. This continues until one car finishes. Each pupil should keep a score sheet similar to the one shown below. These are checked at the end of the game for correctness. To adapt the game to the review of subtraction, markers may be placed at the finish line and moved backward as numbers selected by the spinner are subtracted each time.

### 13. Fill the Squares

The object of this game is to fill the empty spaces according to the pattern established by the numbers already in place. The game may be made more difficult by changing the interval, or by breaking the rhythm pattern.

						8
	$5\frac{1}{2}$					
					$3\frac{1}{2}$	
			$1\frac{1}{2}$			
$\frac{1}{8}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{2}$			$\frac{7}{8}$

### 14. Back and Forth

Divide the class into two teams and line them up facing each other. Start the game by giving the fraction  $1/16$ . The first child of Team A is to give the fraction  $1/16$  larger or  $2/16$ , but in the lowest terms it can be reduced or  $1/8$ . Then, the first member of Team B gives  $3/16$ . This continues back and forth from team to team. When fractions for sixteenths have been completed tenths, eighths, sixths, fifths, fourths, thirds, and halves can be given. A score of one point for each correct answer can be kept. The team with the most points wins.

### 15. We Make One

Write three fractions whose numerators are 1, whose denominators are different and when added together equal 1.

Example:  $1/2 + 1/3 + 1/6 = 1$ ; rename fractions before adding  
 $3/6 + 2/6 + 1/6 = 1$

Then proceed using the same rules with 4, 5, 6, etc. fractions.

### 16. Magic Squares

First find the sum of one row of the square. Then you know what the other rows should add up to. Circle the key fact in each addition. Then add.

$7\frac{5}{6}$	$4\frac{2}{9}$	$3\frac{1}{3}$
	$8\frac{5}{8}$	
$5\frac{1}{18}$		

Sum is \_\_\_\_\_

$6\frac{1}{8}$	$5\frac{2}{3}$	$4\frac{5}{6}$
$7\frac{1}{12}$		
		$3\frac{7}{12}$

Sum is \_\_\_\_\_



### 17. Postman Game

Prepare envelopes with an address on each like  $\frac{1}{4}$  of 4 = ?,  $\frac{1}{3}$  of 15 = ?, and etc. Each desk represents a residence and is numbered accordingly. The postman delivers his letters to the proper address determined by what ? is. If the occupant discovers an error, he reports to the postmaster and if he can correct the error, he continues until the letters are all distributed.

### 18. Common Denominator Game

Cut 20 or more tagboard flashcards. On each card write two or three unlike fractions. Place the common denominator on the back of each card. This game can be adapted for individual, small, or large group use. In group use, the teacher or a leader can hold up each card in turn and the group can respond on paper.

### 19. Complete The Chart

factors	$\frac{2}{3} \times \frac{4}{5}$	$\frac{3}{4} \times \frac{1}{2}$	$\frac{2}{3} \times \frac{2}{5}$	$\frac{4}{5} \times \frac{3}{5}$	$\frac{2}{5} \times \frac{2}{7}$	
product of numerator	8					
product of denominator	15					
total product	$\frac{8}{15}$					

### 20. Fill in the Spaces - One has been done for you.

X	$\frac{2}{3}$	$\frac{4}{5}$	$\frac{1}{3}$	$\frac{1}{2}$
$\frac{1}{2}$	$\frac{1}{3}$			
$\frac{3}{4}$				
$\frac{1}{8}$				
$\frac{5}{6}$				

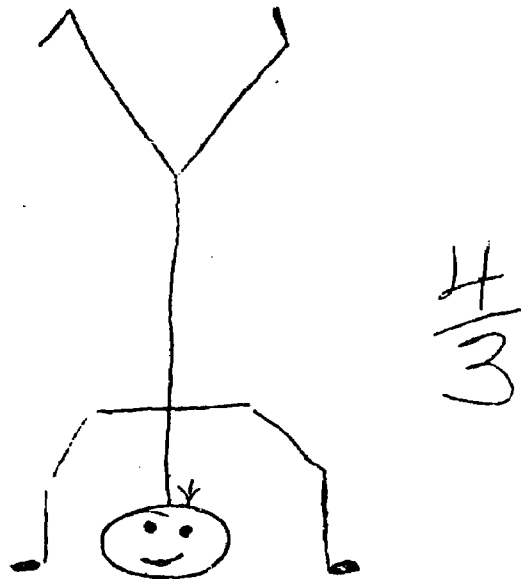
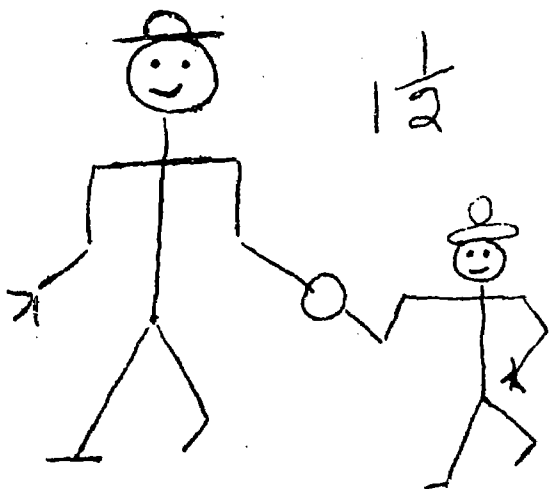
### 21. Fruit Punch (serves 10)

2  $\frac{1}{4}$  cups grape juice (5  $\frac{5}{8}$  ans)  
 1  $\frac{3}{4}$  cups orange juice (4  $\frac{3}{8}$ )  
 1  $\frac{1}{2}$  cup pineapple juice (3  $\frac{3}{4}$ )

3  $\frac{3}{4}$  cups water (9  $\frac{3}{8}$ )  
 1  $\frac{1}{4}$  cup lemon juice ( $\frac{5}{8}$ )  
 $\frac{3}{4}$  cup sugar (1  $\frac{7}{8}$ )

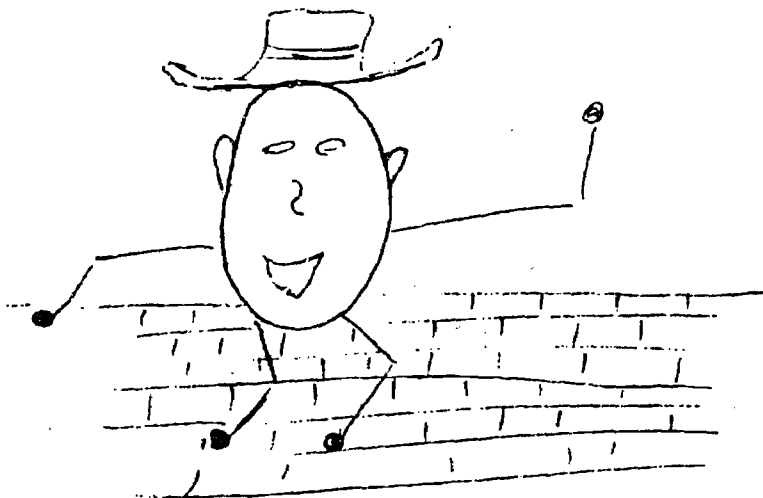
If you wish to serve a class of 25, you would have to multiply each ingredient by 2  $\frac{1}{2}$ . Recopy the recipe showing the new amounts.

22. Bulletin Board Suggestion

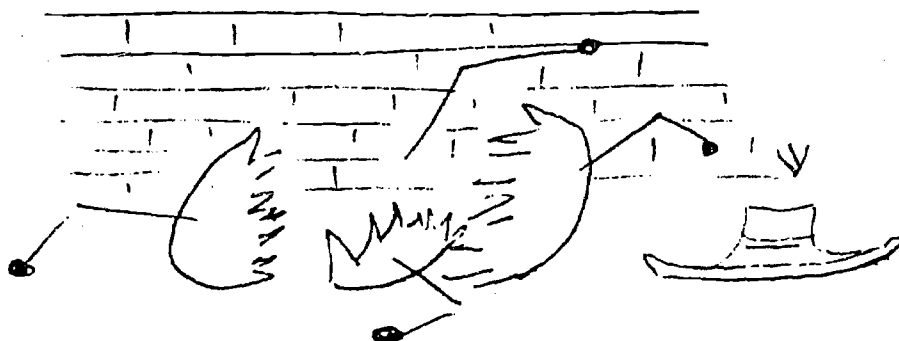


23. Bulletin Board Suggestion

Humpty Dumpty sat on the wall.



Humpty Dumpty had a great fall.



WHAT A FRACTIONAL EXPERIENCE!!

# USING DECIMALS TO NAME RATIONAL NUMBERS

## II. Numeration and Number Theory

S-5, R-6

1. Given dollars and cents expressions, such as \$2.50, the learner can express the amount as a mixed number, in its lowest terms.

Readiness: Experience with money.

- 1a. Place examples on the chalkboard such as:

$$\$3.25 = 3\frac{1}{4}$$

$$\$4.50 =$$

$$\$2.75 =$$

$$\$5.20 =$$

(Express the rest of the sums of money as mixed numbers. Add to this list for the students to complete.)

- b. Have each child draw a number line. Point out that you can use two ways to name each number and that each number on the line is named by a fraction as well as by a decimal.

Illustration:  $\frac{0}{10}, \frac{1}{10}, \frac{2}{10}, \frac{3}{10}, \frac{4}{10}, \frac{5}{10}, \frac{6}{10}, \frac{7}{10}, \frac{8}{10}$   
0 .1 .2 .3 .4 .5 .6 .7 .8

- c. See activities number 1. and 2.

2. The learner can write in decimal notation any ten numerals made up of tenths only, and of whole numbers and tenths, in random order, dictated by the teacher. The learner can, in addition, read back what he has written if asked to do so.

- 2a. Each child lists five decimal numbers in order by size. ( Example: 1.4, 3.6, .1, .5, 6.8). Choose a child to write his numbers on the chalkboard in random order. The other children are to copy the numbers in order by size. The first child called on who has the correct arrangement writes his numbers on the chalkboard in random order.

- b. See activity number 3--- for ODOMETER

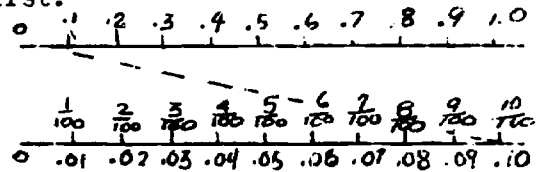
USING DECIMALS TO NAME  
... RATIONAL NUMBERS

II. Numeration and  
Number Theory

S-5, R-6

3. The learner can write in decimal form any ten numerals made up of hundredths only and of whole numbers and hundredths, in random order, dictated by the teacher. The learner can, also, read back what he has written if asked to do so.

- 3a. Draw two number lines, the second being so to speak, a magnified version of the first.



Explain that the key number is ten and that we continue making subdivisions of ten in order to name smaller and smaller numbers.

- b. See ACTIVITY 2(A), do the same thing using HUNDREDTHS. Example: (0.13, 2.22, 0.44, 73.67, 7.01)
- c. See ACTIVITIES 4. and 5.

4. The learner can, rite from dictation a group of fifteen numerals, including tenths, hundredths, and thousandths, some with whole numbers and some without whole numbers.

- 4a. Same type of activity can be done as in ACTIVITY 2(A). Examples should include tenths, hundredths and thousandths, some with whole numbers and some without.

- b. See ACTIVITIES, 6, 7, and 8.

5. Dictation of numerals, extended to twenty to be written by the student including tenths, hundredths, thousandths, ten-thousandths, and hundred-thousandths, with and without a whole number.

- 5a. Use OBJECTIVE 5 as an evaluation.
- b. See ACTIVITY number 9.

USING DECIMALS TO NAME RATIONAL NUMBERS

II. Numeration and  
Number Theory

S-6, R-7

6. Given a list of ten decimal notations which contain a mixture of four-, five-, and six-place decimals, some in combination with a whole number and some without, and beside each of which is written "ten-thousandths - hundred-thousandths - millionths -" the learner can circle after each notation which of the three denominators listed would be used in reading the decimal. In addition, the learner can read any of the given decimals orally if asked to do so.

- 6a. See ACTIVITY numbers 10 and 11.

USING DECIMALS TO PERFORM THE FUNCTIONS OF  
ADDITION, SUBTRACTION, MULTIPLICATION AND  
DIVISION OF RATIONAL NUMBERS

III. Real Number  
Operations

S-6, R-7

7. Given ten examples, five addition and five subtraction with both examples containing decimals of varying denominations from tenths to hundred-thousandths, and with the addition examples having from four to seven addends, the learner can perform the indicated operation and can read the resulting sum or difference orally if asked to do so.

Readiness: Can use decimals to name rational numbers.

- 7a. See ACTIVITY number 12.  
b. See FOLLOW-UP, ADDISON-WESLEY, Teacher's Edition, 1968, p 223.

8. When given ten examples including five multiplication and five division, which require that a decimal be divided by a whole number, and such that there is a mixture of 3-, 4-, and five-place decimals, the learner can perform the indicated operation, then read his product or quotient if called upon to do so.

9. Using decimal divisors, including tenths through hundred-thousandths, the learner can divide these decimal divisors into the indicated whole number dividend, in a given list of ten examples.

8a. See ACTIVITIES 13, 14, and 15.

b. Scientific notation makes it easy to handle very large numbers, like those used to measure distances in space. The students may also, be interested in converting to scientific notation some extremely great distances, like the distance to Andromeda, a large galaxy that is 9 billion miles from earth. ( See Addison Wesley, Book 6, (1968) p 245

9a. Problems can be selected in division giving the students practice using decimal divisors. Examples:

$$1.5 \overline{)24}$$

$$0.13 \overline{)0.0169}$$

$$0.06 \overline{)36}$$

$$0.026 \overline{)0.39}$$

$$0.00008 \overline{)40}$$

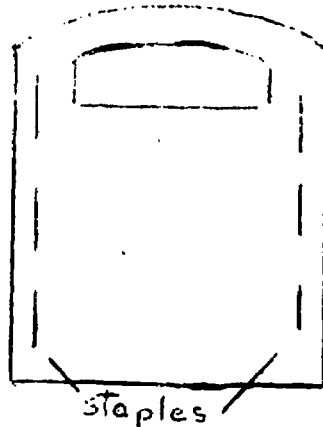
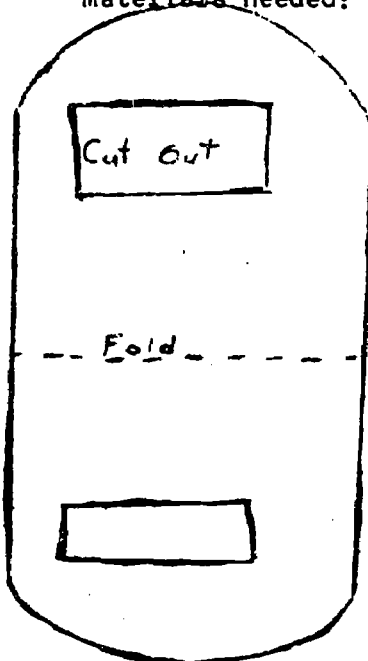
$$0.007 \overline{)0.00049}$$

## GAMES FOR USE WITH DECIMALS

### 1. SHOPPING:

Number of players: Two to twelve.

Materials needed: Play money for each child in sufficient number and variety that he can show any amount from 1¢ to \$1.00 (minimum: 4 pennies, 1 nickel, 2 dimes, 1 quarter, 1 half-dollar) and purses (can be made of butcher paper something like one shown). Also needed are flash cards with pictures of commodities and prices.



Procedure: Leader or teacher holds up flash card. Players arrange correct amount on their desks or tables. Leader says "sold" to those who show correct amounts.

### 2. DECIMALS; STORE

Number of players: two to eighteen.

Materials needed: Large sheets with pictures of articles, each one marked with a price tag, (or pictures may be drawn on the blackboard). Each child needs paper, pencil, and possibly crayons.

Procedure: Each child starts with an imaginary amount of money which he writes at the top of his paper. With this he "buys" things by drawing a picture of the item desired in one square on his paper. He "pays" for it by subtracting the price from the amount of money with which he started. The money left over is written on one side of the next square, and other purchases are made (items drawn in and prices subtracted) until the entire amount is spent.

# ODOMETER CONSTRUCTION

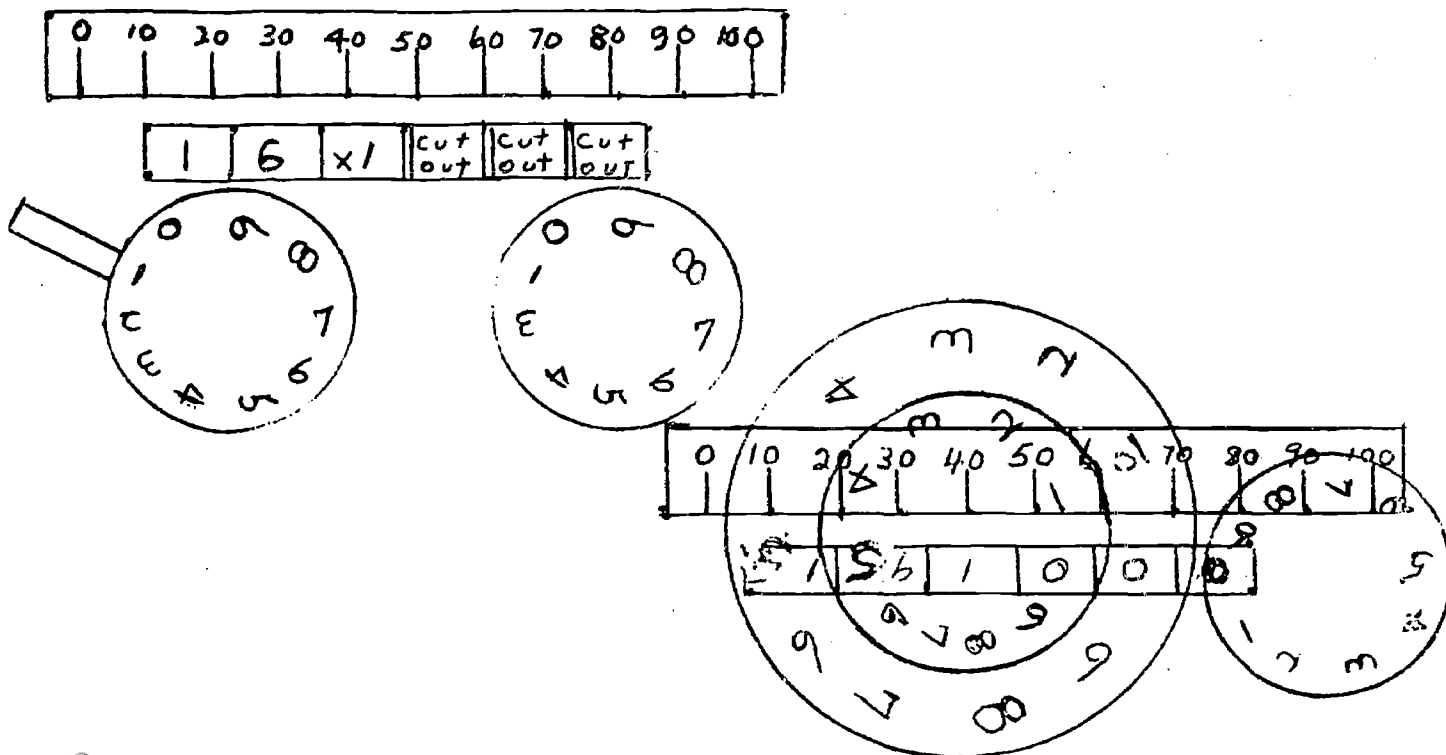
3. Materials: Ditto master, 10"x12" tagboard, crayons, and twopronged brass fasteners.

The odometer should be made in quantities large enough to enable each child in a classroom to have one as decimal fractions are studied. Make the pattern for the odometer body and the two smaller disks on one ditto master. The body of the odometer is  $10 \frac{3}{4}$  inches long and 3 inches wide. The speedometer scale is  $8 \frac{3}{4}$  inches long and  $1 \frac{1}{4}$  inches wide. The space for each numeral in the tens, ones, and tenths places is 1" by  $\frac{3}{4}$ ". The disks showing tens and tenths are each 3" in diameter. The tab on the tens disk is used to turn the disk after it is attached to the odometer.

The patterns for the ones disk should be put on another ditto master. There is room for three such disks on one ditto master. This wheel has a diameter of  $4 \frac{3}{4}$ ". After the ditto masters have been prepared, the patterns for the parts can be put on the tagboard by hand-feeding the tagboard sheets through the ditto machine.

Each child can be given the parts he needs to assemble his odometer. After cutting all of the parts out, the numerals should be colored. The numerals for the tenths place should be black.

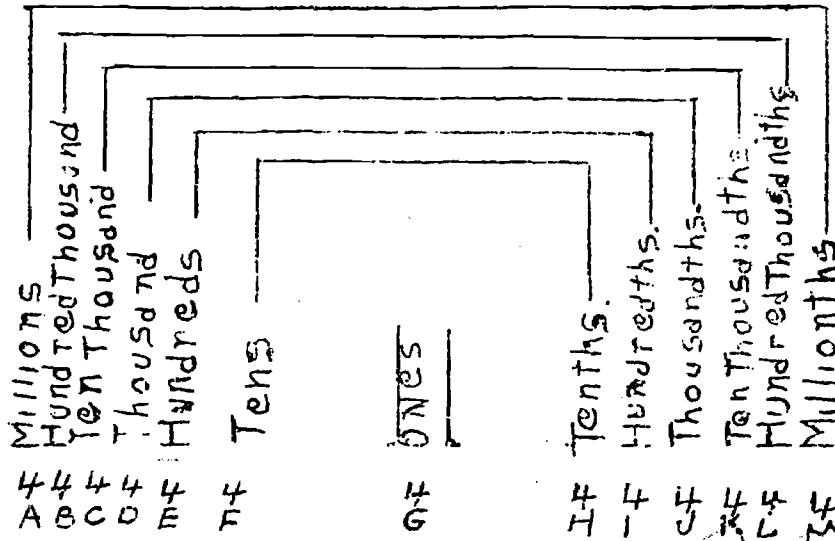
The teacher's odometer should be much larger. It can be made from railroad board or any heavier tagboard.



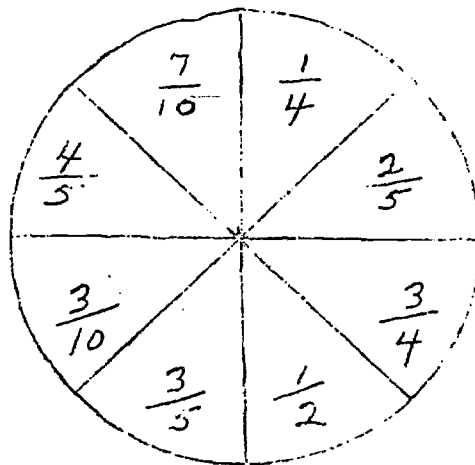


## DECIMALS

4. **DECIMAL PLACE VALUE-** A place value chart shows places with similar names, such as tens and tenths, in proper relationship. Each child should prepare his own place value chart so he can better understand the place value of numbers. It should be explained that 4 above G means 10 times as much as 4 above H,  $\frac{1}{10}$  as much as 4 above F,  $\frac{1}{100}$  as much as 4 above E, etc. The concepts of 10 times as much etc. apply to the left of the decimal point; the concepts of  $\frac{1}{10}$  as much, etc. apply to the right of the decimal point.



5. **DRIVE THE DECIMAL WHEEL:** Duplicate a wheel like the one shown below. The children are to go around the wheel as fast as they can, writing decimal names, on the outside of the circle, for the fractions. Also, fraction names can be written for decimals or mixed numbers and improper fractions can be used. Only fractions which are familiar with the children should be used.



## DECIMALS

6. **DEVELOP YOUR DECIMALS:** Have pupils go to the board or use paper, slates, etc., and draw some figure such as a jack-o-lantern. Then give several decimal numbers to put into the figure. Say "Go" and the pupil should add all the numbers as fast as he can. Time him and record his score on the chalkboard under his name. The next child takes his turn. The child who gets his total correct in the shortest amount of time wins. Also, this may be done with subtraction. A number in the stem of the pumpkin is subtracted from every other number in it.



7. **DECIMAL CHART AND SLATED CLOTH:** Cut a square out of slated cloth 27"x27" with pinking shears to give it an edge that will not ravel. Draw a 20" square in the center and divide it into 100 squares, outlined in white enamel. Use white or colored chalk to shade the tenths, hundredths, or any part of the whole to be illustrated. A blackboard eraser will erase any work or it is also washable. A washable white curtain shade outlined with black crayon can be used instead of the slated cloth.

NOTE: This can also be done with graph paper or plain paper marked off with a sewing machine.

8. **SPOOLS:** Thread 100 spools, identical in size and shape, on a heavy cord. Paint in red with poster paint or airplane dope every tenth spool to facilitate counting. Cut spools with a power saw or by hand to show decimal equivalents. For example, the thirteenth spool can be cut in halves so that  $12 \frac{1}{2}$  spools may be separated from the rest to show  $\frac{1}{8}$  of the whole. Make a number of dividers by cutting a 4" cardboard circle to the center and slipping it over the string in between the spools.

9. DECO: Draw a playing area similar to the one shown below on the board and divide the class into teams. The members of each team takes turns filling the boxes in their column with decimals that meet the requirements of the directions common to both teams. Decimals may be corrected by the captain of each team as he discovers errors or as they are pointed out to him by his own team members. If the opposing team discovers an error, it and another decimal must be erased and the boxes left empty. At the end of the game the teacher erases any errors or duplications, even if a duplication appears in Team A and Team B's area. The team with the most decimals left in its area wins. The playing areas may also be dittoed for use by pairs of pupils. After each has filled his own area, he crosses out all errors made by the opposition.

TEAM A    TEAM B	
Write decimals with 9 in tenths place	
1	1
2	2
3	3
4	4
Write decimals having no hundredths	
5	5
6	6
7	7
8	8

10. NUMBER BLOCKS: Duplicate a sheet that looks like the one in the illustration below, or a similar one. On the back of it duplicate numbers or letters in sequence, one to each square for checking purposes. Cut out the squares, shuffle them, and pass them to pupils in sets. Have each child recreate the original rectangle by arranging the numbers in order from smallest or largest by rows. Then turn over the squares to see if the sequence of numbers is correct.

0.002	0.5%	0.02	0.09
9 1/2%	38%	0.4	0.96
98%	100%	1.01	1.211
175%	1.8	2	2.03%

4	3	2	1
8	7	6	5
12	11	10	9
16	15	14	13

11. DECIMAL RELAY: Write two sets of fractions in columns on the board. Put the same fractions in each column but in different order. Divide the class into teams. When the signal is given to start, the first member on each team goes to the board and writes the decimal equivalent beside the first fraction in his team's column. As soon as he returns to his seat, the next member goes up and so on. The team which writes the most correct decimal equivalents wins.

## DECIMALS

### 12. DECIMAL RACE:

Number of players: Two teams of five to ten members each.

Materials needed: A collection of large flash cards with three or four decimal-fractions on each one.

7.5
1.5
.10

.0085
.200
.25

Procedure: This game could take the form of a race. There could be 10 laps to the race, and the team that answers 10 problems correctly first wins the race and the game. Teams supply individual contestants in rotation. If the contestant identifies the decimal that is the largest, his team advances one lap towards the goal of 10. If one contestant misses, the contestant of the opposing team has an opportunity to give the correct answer. If this second student misses, the first team has an opportunity to answer correctly, etc. This is continued until the correct answer is given. Student leaders write down the answers on the board as given by the contestants. They draw an "X" through any incorrect answers. They also keep a record of the laps completed.

### 13. MATCHING:

Number of players: Two to eighteen.

Materials needed: Cards with decimal facts on them and room to match the answers as illustrated, diagram "A". Numerous small squares of tagboard containing the answers as shown on diagram "B".

A

$6.24 + .2$	
$1.50 \div 5 =$	
$1.0074 - 1 =$	

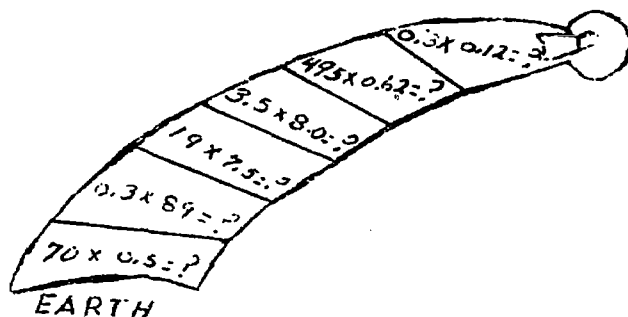
B

.30
6.44
.0074

Procedure: Give each child a card and the mixed-up answers. (Have as many different cards as there are children in each row, so that at the end of each game the cards can be passed to another child and the game played as before.) The object of the game is to match correct answers to the problems. If the game is played by individuals, the first one to finish is the winner. If played by teams or rows, the first child finished in each row stands.

## DECIMALS

14. Show the flight of a rocket with division marks in which are written examples involving decimals, as shown below. Make out a set of small cards with several duplications written on them of each of the examples in the rocket's flight. Write the correct answer on the back of each card. Give each of 3 or 4 students a marker which is placed on the "earth" at the start of the game. Have the students shuffle and pass out the cards, placing them example side up in front of them and reading in turn each card. If a card is read that matches the one in the space in front of his marker, he may advance. If he can give a reasonable estimate for the example, The first to reach the satellite wins.



15. WHO SHOULD PAY THE CHECK? Ask a friend to count all of the loose change in his pocket. Tell him you will be able to find how much change he has if he just shows you the final answer to these steps.

Multiply the amount by 2	$2(64) = 128$
Add 3 to the product	$128 + 3 = 131$
Multiply the sum by 5	$5(131) = 655$
Subtract 6	$655 - 6 = 649$

Then ask him to show you the final answer, 649, and you can tell him immediately that the change is 64 cents. Just cross out the ones digit, 64~~9~~, and the remaining digits express the change.

### ALGEBRAIC PROOF:

Start with change	C
Multiply by 2	2C
Add 3	$2C + 3$
Multiply by 5	$10C + 15$
Subtract 6	$10C + 9$

Addison-Wesley Mathematics Program

Book 5 (1964) pp 270-285 (1965) Workbook pp. 84-87  
Book 5 (1968) pp. 261-271 (1968) Workbook pp. 87-90  
Duplicator Masters (1969) pp. 50-51 (Addition and Subtraction)  
Possible Activities: Book 5, Addison-Wesley (1968) Teachers Edition:  
pp. 256-257, 264-265; Follow-up: pp. 264-265; Cumulative review: pp. 268-269  
Evaluation: pp. 270-271. Also end of chapter test. (85% on test recommended).

Seeing Through Arithmetic, Scott Foresman Co.

Book 5 (1963) pp. 254-257, 564-565.

Addison-Wesley Mathematics Program

Book 6 (1964) pp. 228-261 (1965) Workbook pp. 72-84  
Book 6 (1968) pp 216-249 (1968) Workbook pp. 76-88  
Duplicator Masters (Now available from company)  
Possible Activities: Book 6 (1968) Teachers Edition: pp 216-217, 230-231,  
234-235, 240-241, 242-243, and 245.  
Follow-up: pp. 217, 219, 223, 228, 229, 235, 243, 246, 247, 249.  
Evaluation: End of chapter test

Seeing Through Arithmetic, Scott Foresman Co.

Book 6 (1963) pp. 169-182, 216-246

Manipulative Devices

Transparencies:

Charts: (Aladdin) No. 529 Equivalent Chart, No. 762 Decimal Place Value Chart.

\*Liquid Duplicator Masters (Continental Press Inc.)

Ginn Arithme-sticks (Taylor School) Milton Bradley Co.

Abacus

\* L.W. Singer Co., "Packaged Assistant" for the busy teacher kit

\*S.R.A. or Imperial Drills tapes.

(\*Material that is not available in the system, suggest these might be ordered  
in the near future)

Field Educational Publications, Inc., Cyclo Teacher, Mathematics Cycles.

M-81 Understanding Decimals

M-82 Using Zero in Decimals

M-85 Multiplying Decimals

M-86 Changing Decimals to Fractions

M-87 Dividing Decimals

M-88 Dividing by Decimals

M-89 Changing Fractions to Decimals

## Filmstrips.

- 340 Decimal. Whole Numbers  
341 Decimal Fractions, Tenths  
342 Decimal Fractions, Hundredths  
343 Adding Mixed Decimals  
344 Decimal Fractions, Subtraction  
345 Decimal Fractions, Multiplication  
346 Decimal Fractions, Division

**Teachers Reading References:**

Dutton, Wilbur H. and L. J. Adams-- Arithmetic for Teachers, Prentice Hall (1961), pp. 254-277, "Has a fifth grade lesson plan on developing and understanding of decimal fractions."

DeVault, M. Vera- Improving Mathematics Programs, Charles E. Merrill (1966), pp. 86-89, "when a pupil writes a decimal point he should be expressing an idea rather than performing a computational skill."

Osburn, Roger, M. Verr DeVault, Claude C. Boyd, and W. Robert Houston:  
Extending Mathematics Understanding, Charles E. Merrill, (1961), pp. 72-86,  
 "Decimal Notation--Past and Present-- Has been Represented in a Variety of Ways."

## TESTING MEANINGS IN DECIMALS

- Look at each number carefully and if the "1" on the left represents ten times as much as the "1" on the right, put a loop around that number.  
110      11      1.1      0.11      0.0011
- The decimal point marks the spot where whole ones or groups of ones are on the left and parts of one are on the right.  
Yes \_\_\_\_\_ No \_\_\_\_\_
- The decimal point occupies a place the same as the digits do.  
Yes \_\_\_\_\_ No \_\_\_\_\_
- The digits in a decimal number have place value.  
Yes \_\_\_\_\_ No \_\_\_\_\_
- Underline each number in which the figure on the right represents one tenth as much as the figure on the left.  
55      2.2      0.45      0.33      0.0011
- Underline each number if the "2" represents 20 times as much as the "1".  
210      0.521      32.1      201      0.21
- Underline the number if the "5" represents 25 times as much as the "2".  
502      250      5.2      2.3      5.02
- Put a loop around every number that is nearer to 3 than it is to 2.  
2.1      2.6      2.4      2.5      2.9      3.4
- This number 6.8, is read six and eight tenths. That means that the number is:  
(underline one)      8 tenths less than 7      8 tenths more than 6  
                                    8 tenths more than 1      8 more than 60
- If the decimal point is left out of the answer in the example at the right, the answer will be: (check one)      0.6  
Too small \_\_\_\_\_; ten times as large as it should be \_\_\_\_\_; x0.6  
100 times as large as it should be \_\_\_\_\_.

11. Draw a loop around every number that equals one fourth.  
0.25    0.250    25    2.5    0.025
12. Writing a zero after the number 125 makes it represent ten times as much.  
Check the answer that explains why this is so.  
☐ Because there is a zero in ten.  
☒ Because the zero in ones' place makes the other number move one  
 one place to the left, which makes them ten times as large.  
☐ Because 50 is bigger than 5.  
☐ Because 1000 is ten times 100.
13. Writing a zero after the 5 when 5 is 0.5 (5tenths) does not change its value;  
why? Check the right answer.  
☐ Because 50 hundredths = 5 tenths.  
☒ Because the zero did not come between the 5 and the decimal point.  
☐ Because writing zeros after numbers never changes their value.
14. Write the figures after each written numbers:  
 Nine hundred ten thousandths \_\_\_\_\_  
 Nine hundred and ten thousandths \_\_\_\_\_  
 Nine hundred ten thousandths \_\_\_\_\_  
 Three and one-third tenths \_\_\_\_\_  
 Twenty-five hundredths \_\_\_\_\_
15. Decimals may be read in more than one way:  
 2.5 may be read as \_\_\_\_\_ tenths.  
 \$3.75 may be read as \_\_\_\_\_ cents.  
 1.0 may be read as \_\_\_\_\_ tenths.  
 1.25 may be read as \_\_\_\_\_ hundredths.  
 1750 may be read as \_\_\_\_\_ tens.
16. Write the figure after each of the following:  
 One hundred fifteen cents \$ \_\_\_\_\_  
 Twelve dollars and 2 cents \$ \_\_\_\_\_  
 Five and one-half cent \$ \_\_\_\_\_  
 One dime \$ \_\_\_\_\_  
 Two and one-fourth dollars \$ \_\_\_\_\_
17. The carrying or re-grouping in this example 2.6 is the same as 26 because  

$$\begin{array}{r} 2.6 \\ +2.7 \\ \hline \end{array}$$

$$\begin{array}{r} 26 \\ +27 \\ \hline \end{array}$$

 (underline every true statement)  
 Ten of the tenths make "1" in the first example just as ten of the ones make  
 "1" in the second example.  
 Ten of the units in any column make "1" in the next column at the left.  
 The relation between any two adjacent columns is the same.
18. Is this correct?  $0.9 + 0.8 = 0.17$     Yes    No



RELATIONSHIP OF FRACTIONS AND DECIMALS  
TO PERCENT AND VICE VERSA

II. NUMERATION AND  
NUMBER THEORY  
Per Cent  
G6-S, G7-R

1. Given ten expressions of per cent such as 25% the learner can express each per cent as a ratio of the per cent to 100.

Readiness: Test to determine if the child can use decimals to name rational numbers and perform the functions of addition, subtraction, multiplication, and division of rational numbers.

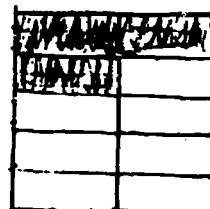
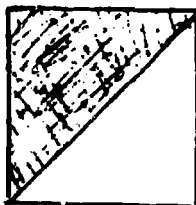
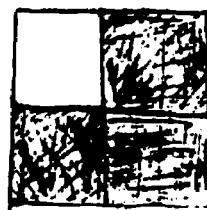
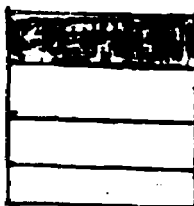
2. The learner can express each of ten given per cents as a ratio to 100 and can, in addition express the percent as a common fraction in its lowest terms.

1a. MATHEMATICAL BACKGROUND

If  $x$  stands for a number,  $x\%$  expresses the ratio of  $x$  to 100. For example, 3% expresses the ratio of 3 to 100. This ratio can be expressed by the fraction  $3/100$ , which in turn can be expressed by the decimal .03. In other words, every rational number that can be expressed as a per cent can also be expressed as a fraction or as a decimal.

- b. See activities #1,2 (Many other comparisons can be used.)

- 2a. Describe the shaded region of each drawing with a fraction, another fraction with a denominator of 100, and a percent.



RELATIONSHIP OF FRACTIONS AND DECIMALS  
TO PER CENT AND VICE VERSA

II. NUMERATION AND  
NUMBER THEORY

G6-S, G7-R

3. When given ten per cents the learner can express the per cent in their correct decimal notation.

- 3a. See activity #3.  
b. Display the following per cents on the chalkboard:  
1% 5% 13% 25% 73% 100%  
Have pupils use the definitions of per cent to change each of the per cents to a fraction which expresses hundredths. Then have pupils express each fraction as a decimal. Inform pupils that in today's lesson they will discover how to change a per cent notation to a decimal without first writing an equivalent fraction.  
c. Have pupils write a paragraph describing how a per cent can be changed to an equivalent decimal and how a decimal can be changed to an equivalent per cent.

SOLVING PER CENT PROBLEMS AS PROPORTIONS

III. REAL NUMBER  
OPERATION  
Per Cent

G6-S, G7-R

4. Given ten word problems for finding a given percent of a number, the learner can state the problem as a proportion having one unknown then proceed to solve for the unknown.

Readiness: To be able to recognize relationship of decimal to per cent and vice versa.

- 4a. Write the following open sentences on the chalkboard.  
 $10\% \times 130 = n$      $32/n = 25/100$   
 $12/30 = n/100$   
Have pupils make up a story problem for each of these open sentences.  
b. See activity number 4.

## SOLVING PER CENT PROBLEMS AS PROPORTIONS

III. REAL NUMBER  
OPERATIONS  
Per Cent  
G6-S, G7-R

5. From a list of ten problems in which you find what per cent one number is of another, the learner can state the problem as a proportion having one unknown, then solve for that unknown.
- 5a. Example: One season Bert pitched in 209 games and won 17 of them. What fractional part of the games that he pitched did Bert win? What per cent of the games did he win?  
 $n \times 20 = 17$  (Equation)  
 $17/20 = n/100$  (Proportion)
- b. See activity #4
6. Given ten word problems, containing different situations in which the solution calls for finding a given per cent of one number, for finding what per cent one number is of another and for finding the whole (100%) when a part is known, the learner can state the problem as a proportion having one unknown, then proceed to solve for the unknown.
- 6a. Example: A men's club wants to present a school with some new books costing \$60. If they sell balloons at a profit of 30% on the balloons what must the total sales be?  
 $30\% \times n = 60$  (Equation)  
 $30/100 = 60/n$  (Proportion)
- b. See activity #4

# SOLVING PER CENT PROBLEMS AS PROPORTIONS

III. REAL NUMBER  
OPERATIONS  
Per Cent  
G6-S, G7-R

## 7. Discount

When the learner is given ten list prices and a discount for each price expressed as a per cent, he can compute the net price after the discount has been applied.

- 7a. A girl's blouse was marked at \$8. If there was a 20% discount what was the selling price of the blouse. (continue proportional method to find discount)

$$20\% \times 8 = n \text{ (Equation)}$$

$$20/100 = n/8 \text{ (Proportion)}$$

$$100n = 160 \quad \$8.00$$

$$n = \$1.60 \quad \underline{-\$1.60}$$

$$\quad \quad \quad \$6.40 \text{ S.P.}$$

- b. See Addison-Wesley (1968) pp. 281

## 8. Interest

Given ten word problems in which the principal, the interest expressed as a percent, and the time stated, the learner can compute the amount of the interest for the time period and the amount of the principal plus interest.

- 8a. Example: Joan borrowed \$250. for one year at 8% interest. How much did she pay the bank at the end of the year?

$$8\% \times \$250 = n \text{ (Equation)}$$

$$8/100 = n/250 \quad \$250 \text{ prin}$$

Reduce  $2/25 = n/250 \quad \underline{\$20} \text{ int}$

$$25n = 500 \quad \$270$$

$$n = \$20 \text{ Int}$$

- b. See Addison-Wesley (1968) p. 282

## ACTIVITIES--PER CENT

### 1. TRANSPARENT OVERLAYS FOR TEACHING PERCENT

Percent is used as a means of expressing rational numbers, as are decimals and common fractions. Children should be impressed with the need for being able to express percent as a decimal or common fraction. Greater emphasis is being placed on the use of percent as a way of expressing a ratio. Children should have opportunities to learn the meaning of percent as used to express ratios. The procedure described here is a means of using transparent overlays in helping children understand the meaning of percent.

#### EXAMPLES:

The ratio of three to five is shown using disks on a magnetic or flannel board. The meaning of this ratio should be discussed to bring out the idea that a ratio expresses a relationship which exists between the members of two sets. An identical set of objects is put on the flannel board next to the first and the ratio of six to ten is shown. The fact that the ratio of three to five is the same as the ratio of six to ten should be discussed. Put a third set of objects representing the same ratio on the flannel board. Again, discuss the meaning of the ratios 3:5, 6:10, and 9:15. The children should be able to discover that the two terms of the ratio three to five can be multiplied by two to give the equal ratio of six to ten. The terms can be multiplied by three to give the equal ratio of nine to 15.

At this point, project a transparency that illustrates the ratio of three to five as first shown on the magnetic or flannel board. The chart at the bottom of the transparency shows the ratio expressed with numerals. Turn an overlay over the first transparency to show the ratio as six to ten. Call attention to the numerals for this ratio on the chart.

Additional overlays should be used to lead the children to see that the ratio can be expressed as 60 to 100. Follow the use of this transparency with its overlays with another showing a second ratio, such as 11 to 20 (11:20) and the way it can be expressed as 55 to 100 (55:100). The fact that ratios can be written in the form  $\frac{55}{100}$  should be brought out as overlays are used. The idea of the proportion should also be brought out. When the ratios 3:5 and 6:10 are discussed, the fact that they can be written to show that  $3:5 = 6:10$ , or  $\frac{3}{5} = \frac{6}{10}$ , should be brought out.

After the overlays have been used in reviewing the meaning of ratios and proportion, the concept of percent should be introduced. The idea that percent is a form of expressing a ratio of one number to another should be brought out.

When solving problems involving percent, the problem should be written as a proportion.

2. Ratios in percent form can be used to compare many things. Complete the following table on the chalkboard as a class activity or ditto for individual use.

Player	Ratio Number of Hits to Times at Bat	Frac- tional Form	Denomi- nator 100	Decimal Form	Per Cent Form
Shortstop	6 out of 20				
First Baseman	7 out of 25				
Catcher	3 out of 10				
First Pitcher	1 out of 5				
Second Pitcher	1 out of 4				
Third Pitcher	0 out of 2				
Center Fielder	7 out of 20				

3. Help in emphasizing interrelationships of fractions, decimals, and percentages.
- Write a percentage, fraction, or decimal on the chalkboard (Example: .7)
  - Children are to write the two alternative forms of what is on the chalkboard (Example:  $\frac{7}{10}$ , 70%)
  - The first child chosen who has the correct answer may place the next percentage, fraction, or decimal on the chalkboard.

4. Aid in Per Cent

After percentage has been introduced and the pupils have had enough isolated facts on which to build experiences, they collect clippings pertaining to percentage from newspapers, magazines, advertisements, and circulars. Interpretations of these materials are made, and the clippings are used in the following way.

A clipping is chosen and is pasted on notebook paper. Then an original problem is written for it, and solved. To accompany this, the pupils also choose a similar problem from their text and solve it. When the notebooks are completed they show problems pertaining to commission, discount, interest, and so on.

Since the material available for this project is abundant, a limit has to be set on the number of clippings to be utilized. Each pupil selects one clipping for his problem and is allowed an extra page for any others related to sports. In discount, each must have three types of clippings and their accompanying problems, which deal with percentage of discount, fraction off, and regular reduced price.

## MEDIA

1. Addison-Wesley Mathematics Program  
Book 6 (1964) pp. 264-281 (1968) pp. 272-289  
Workbook (1965) pp. 85-89 (1968) pp. 97-101  
Duplicator Masters (1969)  
Teachers' Edition (1968) Activities and follow up pp. 272-289  
Evaluation: Chapter reviews (1964) pp. 280-289, (1968) pp. 288-289  
End of chapter tests with 87% accuracy.
2. Seeing Through Arithmetic Scott-Foresman Co.  
Book 6 (1963) pp. 126-142; 244-246
3. Transparencies: See transparency accompanying activity #1.
4. Charts: Equivalent chart (Aladdin) number 529.
5. "Packaged Assistant" for the busy teacher (L.W. Singer Co.)
6. Buchnell Mathematics, Self-Study System Number 1 (Webster Division McGraw-Hill Co.)
7. Teachers' Reading References:  
Dutton, Wilbur H., and L.J. Adams: Arithmetic for Teachers, (1961)  
pp. 295-319. Has a sixth grade lesson plan for extending the meaning of percent.
8. Nelsen, Jeanne: "Per Cent: A Rational Number or a Ratio" Arithmetic Teacher, Vol. 16, number 2, pp. 105-109 (1969) To clear up inconsistencies in our teaching of per cent.

LINEAR AND METRIC MEASUREMENT  
INCH - CENTIMETER

IV. MEASUREMENT

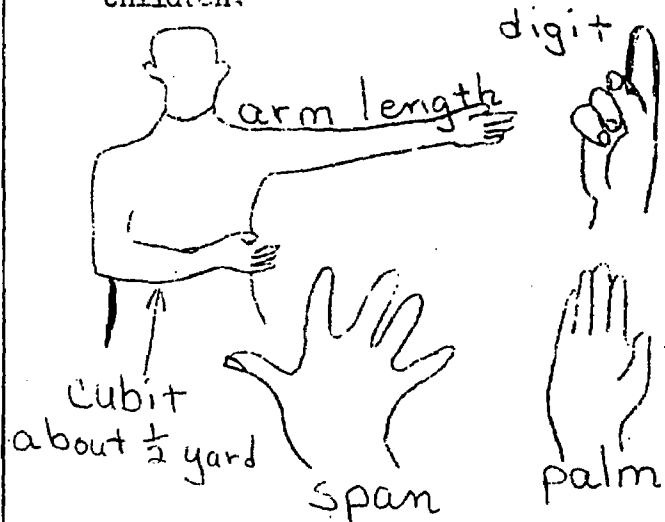
K-1, G1-I, G2-I, G3-S

1. Learner, by using any measure that suitably comes to hand, will be able to make such comparisons as:

longer than  
shorter than  
higher or taller than  
near and far

The objective is to extend these experiences toward understanding the need for standard units of measure and finally to use these with reasonable efficiency.

- 1a. In early times measurements were calculated from parts of the human body—limb measures. The following are some which can be used by young children.



- b. The teacher can give, orally or written on cards for children who can read, such activities as:
1. Measure your neck, wrist and waist. What will you use to do this? How many wrist measurements go around your neck? (Estimate first)
  2. See if your measurements are different from your partner's.
- c. Use your hand span, pace, or length or your foot to measure:
- the length of your desk
  - a piece of paper
  - a book
  - a window pane
  - a chalkboard

Think about the best unit to use before you begin. Compare results with your partners. Are they different? Why?



LINEAR AND METRIC MEASUREMENT  
INCH - CENTIMETER

IV. MEASUREMENT

K-1, G1-I, G2-I, G3-C

1. Continued

- ld. What human units would you use to measure:
  - the height of a horse
  - a mouse
- e. Children may also use arbitrary units which are convenient such as a piece of string, paper, strides, etc. Be sure they compare results.
- f. This preparatory work will hopefully lead children to realize the need for standard units. Here the ruler--inch and centimeter and the yard may be used.
- g. Materials prepared by the teacher--paper pinned to the wall, lines on chalkboard and floor, etc. can now be measured and results compared to show value of standard units.
- h. Addison-Wesley:
  - Book 1--pp. 307-310
  - Book 2--pp. 281-284
  - Book 3--pp. 1-15
  - Book 3-Revised Edition--pp. 164-177
- i. See activity #1.
- j. S.R.A.--Grade 2--pp. 203-205, Linear activities.
- k. Scott Foresman--Book 1, Grade 1--pp. 225-226
- l. Scott Foresman--Book 3--pp. 86-88
- m. Suggested reading for teacher help, McGraw-Hill, Teaching Elementary School Mathematics for Understanding pp. 383-389.
- n. See Recommended List, Item 4.
- o. See activity #22.
- p. Evaluation may be done through teaching observation.

LINEAR AND METRIC MEASUREMENT  
INCH, FOOT, YARD, CENTIMETER

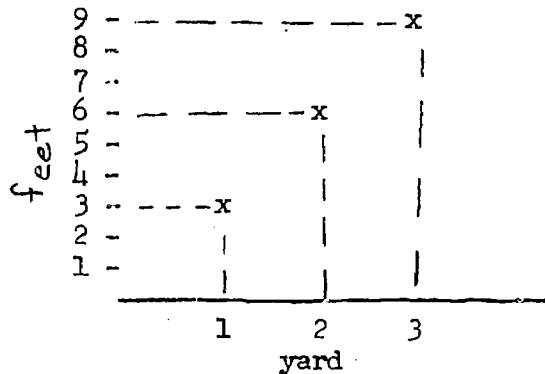
IV. MEASUREMENT

G4-S, G5-R, G6-R

2. Learner can, when asked, measure various lengths in terms of inches, feet, yards and centimeters with 85% efficiency and gain an intuitive understanding of mile.

Readiness: Learner is familiar with inch, foot, yard and centimeter as standard units of measure.

- 2a. Written instructions for individual work might be somewhat as follows:



A simple graph can be made showing conversion of feet and yards. The discovery of zero (0) can be made.

- b. Write down things in the room that you think are about one foot in length.

Objects estimated as 1 foot long	Measured length to nearest inch	Error to nearest inch
Window pane	9 inches	3 inches

How many feet in one yard? Write down how you found out. How long is one of your strides? (to nearest inch) Mark a yard on the floor. How much shorter is one of your strides? (to nearest inch) Measure the length of the classroom by striding. Now use a ruler (which one?) and work out your error.

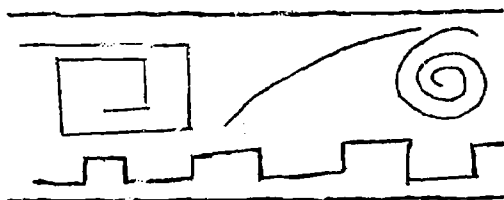
LINEAR AND METRIC MEASUREMENT  
INCH, FOOT, YARD, CENTIMETER

IV. MEASUREMENT

G4-S, G5-R, G6-R

2. Continued

- 2c. Get a large piece of paper and draw lines like these, much bigger, with a felt pen or crayon. Measure the lines. Think how you can do this. Estimate first.



d.

What is the length of your foot?  
Draw around it on a piece of paper like this



Draw around other children's feet.  
Try to find the longest and shortest in class.

If a man's foot is a foot long,  
how much shorter is yours?

- e. Measure your height (how?). Measure your partner's height. What is the difference between heights? If you stood on your partner's head what would your combined heights be?
- f. Addison-Wesley:  
Book 4-pp. 1-7, 23  
Book 5-Measurement, Lesson 1-p. 18  
Lesson 8-pp. 236 & 237  
Book 6-pp. 20-21
- g. See activity #1
- h. See activity #12 & 13
- i. See Recommended List, Items 2 & 3
- j. Activity #16, 23
- k. Evaluation - Complete the following:
- |                      |                       |
|----------------------|-----------------------|
| 48 in = <u>  </u> ft | 2 mi = <u>  </u> ft   |
| 6 yd = <u>  </u> ft  | 2 mi = <u>  </u> yd   |
| 33 ft = <u>  </u> yd | 5 yd = <u>  </u> in   |
| 33 ft = <u>  </u> in | 288 in = <u>  </u> yd |

LINEAR AND METRIC MEASUREMENT  
INCH, FOOT, YARD, CENTIMETER

IV. MEASUREMENT

G4-S, G5-R, G6-R

3. Learner can gain an intuitive understanding of the meter and its prefixes.

- 3a. Addison-Wesley  
Book 5-p. 284  
Book 6-pp. 262-263  
b. Use of centimeter rulers and meter sticks  
c. Suggested reading for teacher help, McGraw-Hill, Teaching Elementary School Mathematics for Understanding-pp. 383-389  
d. History of the metric system, see The World of Measurement, Webster Publishing Co.-pp. 15-19

LINEAR AND METRIC MEASUREMENT  
PERIMETER

IV. MEASUREMENT

G4-S, G5-R, G6-R

4. Learner can, when asked, measure and/or compute the distance around polygons with 85% accuracy.

Readiness: Learner can measure with some accuracy varying lengths in inches, feet, and yards, meter and centimeters.

- 4a. Addison-Wesley:  
Book 4-pp. 14-17  
Book 5-pp. 50-51  
Book 6-pp. 20-21, 239  
b. See Recommended List, Item 3  
c. Additional practice, or testing, Scott Foresman-Book 5-pp. 107-109

LINEAR MEASUREMENT  
CIRCUMFERENCE

IV. MEASUREMENT

G6-S

5. Learner can compute the circumference of a circle with 85% accuracy.

Readiness: Learner is familiar with computing perimeter of polygons.

- 5a. Addison-Wesley-Book 6-pp. 282-283

- b. Another way to find circumference:  
Cut out a circular piece of paper with a radius of one inch. Mark a starting point on a circle and on a line segment.

Then roll the outer edges of the circle along the line segment until the starting point is reached. Then measure the distance between the points. This is circumference. The same can be done with larger circles. To prove this use formula:

$$c = \pi \times 2 \times r$$

- c. Evaluation

Using the formula,  $c = \pi D$ , have the pupils compute circumferences for given diameter. Example:

(a) 6 in (b) 4 ft (c) 8 cm

Find the circumference of the circles with the diameters given above.

MEASUREMENT - AREA

IV. MEASUREMENT

G3-I, G4-I, G5-S, G6-R

6. Learner can find or compute the number of squares or square units in a two-dimensional drawing with 85% accuracy.

Readiness: Developed in Grade 3 when learner is able to count the number of squares in a two-dimensional drawing.

- 6a. Addison-Wesley

Book 3-pp. 1-5, 16-19

Determine area by counting the squares in the picture.

Book 4-pp. 1-3, 8-11

Short cuts for finding area used, but no formula

Book 5-pp. 72-73, develop  $L \times W = A$

Book 6-pp. 42-43

- b. Make a square foot out of cardboard, dividing it into 144 sq in using black lettering ink.
- c. Make a square yard out of cardboard, divide into 9 sq ft and alternate colors so each square stands out. Hang on wall.
- d. See Recommended List, Item 3

7. Learner will be able to find or compute surface area of three-dimensional objects with 85% accuracy.

Readiness: Ability to find area of plane figures will prepare learner for objective two.

- 7a. Addison-Wesley

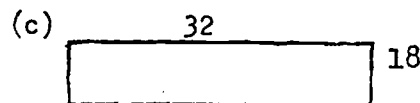
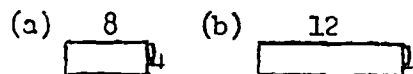
Book 4-pp. 18-21-no formulas used

Book 5-pp. 84-85-use  $A = L \times W$  to arrive at surface area

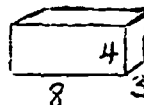
Book 6-pp. 42-43

- b. Evaluation

Find the area of the following:



Find the surface area of the following:



# MEASUREMENT - AREA OF TRIANGLE

## IV. MEASUREMENT

G6-S

8. Learner can find the area of a triangle using the formula  $A = 1/2 BH$  with 85% accuracy.

Readiness: Learner has become familiar with finding area of rectangles using a formula.

9. Learner will discover that the two legs of a right triangle can be used in the formula  $A = 1/2 BH$  in applying the Pythagorean Theorem. No particular mastery is indicated here.

- 8a. Addison-Wesley-Book 6-pp. 216-217

- b. Evaluation

Given several examples of triangles with the base and height, the learner should use  $A = 1/2 BH$ .

Example:  $B = 8$   $H = 6$

$$A = 1/2(8 \times 6)$$

$$A = 1/2 \times 48$$

$$A = 24$$

- 9a. Addison-Wesley-Book 6-pp. 226-227

- b. Suggested reading for the teacher: The Pythagorean Theorem, Webster Publishing Co.

MEASUREMENT - AREA OF A CIRCLE

IV. MEASUREMENT

G6-I

10. Learner will compute the area of a circle through the discovery of the formula  $A = \pi r^2$ . No particular degree of accuracy need be required.

Readiness: Learner has become familiar with finding area of rectangles and triangles through the use of formulas.

- 10a. Addison-Wesley-Book 6-pp. 296-297  
 b. See Recommended List, Item 3, for visual aids that can be purchased.  
 c. No degree of evaluation is recommended. However, teacher observation is an evaluative device.

MEASUREMENT - SPACE GEOMETRY

IV. MEASUREMENT

G5-I, G6-I

11. Learner will demonstrate an understanding of space geometry through the construction of a space figure.

Readiness: Learner is aware of three dimensional objects through experiences with area and volume.

- 11a. Addison-Wesley  
 Book 5-pp. 162-165  
 Book 6-pp. 208-212  
 b. Suggested reading for the teacher: Singer, Sets and Numbers, Book 5 Appendix 1, Geometry-pp. 377-395  
 c. See Recommended List - Item 3, for visual aids that can be purchased.  
 d. Evaluation  
 Make a 3-dimensional paper figure (suggestions in Addison-Wesley, Book 6, p. 212)



MEASUREMENT - VOLUME - LIQUID CAPACITY

IV. MEASUREMENT

K-I, G1-I, G2-I

12. Learner will, through practical experiences, gain the intuitive understanding that liquids must be kept in containers, change shape when poured into different containers but that the amount remains constant. No mastery of standard measures should be expected.

Readiness: Sand play, pouring sand into various containers before introducing water play, is less messy.

- 12a. Water play is indicated, with children working as individual or partners.
- b. Liquid containers should be transparent so children can see amounts of water. Using the eye to measure capacity is difficult at best.
- c. Some examples of individual activities follow. Directions can be given by the teacher for the younger children who can't read. Children can write or tell the result.
- (1) You will need some containers and a funnel. Use the funnel to avoid spilling.
- How many cupfuls fill the jug?  
How many spoonfuls fill the cup?  
How many jugfuls fill the bucket?  
How many teaspoonfuls fill the medicine bottle?
- (2) After some practice like the above, standard measures can be used to fill the jugs and buckets. Estimate first, then pour. How many pints can pour into one quart? How many quarts pour into one gallon? How many  $1/2$  pints can pour into one quart?
- If you pour three pints in a gallon measure, how many more pints will you need to fill it?
- (3) Young children should be encouraged also to experiment to find out approximate capacities of familiar vessels such as teapots, pop bottles, liquid-soap containers.
- d. Addison-Wesley  
Book 1-pp. 285-286  
Book 2-pp. 285-286
- e. See activity 2
- f. Evaluation must be through teacher observation.

# MEASUREMENTS - VOLUME - LIQUID CAPACITY

## IV. MEASUREMENT

G3-S, G4-R, G5-R, G6-R

13. Learner can tell or show how most familiar liquid measures are interrelated and solve problems involving such measures with 85% efficiency.

Readiness: Experience with measures through sand and water play.

- 13a. Materials for measuring will still be necessary for the younger children or those who though older, may still need the practical experience.

- b. Examples for individual projects follow:

- (1) How many school milk containers can you fill from a pint container? A quart container? A gallon?

- (2) Complete this table:

gal	1	2	3	4	5	6	...
qt	4	8		16	24		...

qt	1	2	3	4	5	6	...
pt	2		8	12			...

- c. Addison-Wesley

Book 3-pp. 22-23

Books 4, 5, 6, contain materials scattered throughout

- d. See activities 2, 3, 4

- e. Sample evaluation

4 qt = \_\_\_ gal    16 pt = \_\_\_ gal  
1 gal = \_\_\_ pt

14. Learner can gain an intuitive understanding of the liter as it relates to the quart.

- 14a. Addison-Wesley-Book 6-pp. 262-263

MEASUREMENT - VOLUME - CUBIC UNITS

IV. MEASUREMENT

G3-I, G4-I, G5-S, G6-R

15. The learner, when given a rectangular solid, can give the number of cubic units contained therein with 85% efficiency.

Readiness: The learner is familiar with the inch and centimeter ruler.

16. The learner can gain an intuitive feeling for three-dimensional objects (suggested for grade 6)

- 15a. Addison-Wesley

Book 3-pp. 2-5;20

Book 4-pp. 1-3; 22-23, 21

Book 5-pp. 101, 114-115

Book 6-pp. 84-85

- b. See activities 3 and 27

- c. Evaluation

Addison-Wesley-Workbook 6, 2nd ed.,  
p. 95

- 16a. Addison-Wesley-Book 6-pp. 208-212

MEASUREMENT - WEIGHT

IV. MEASUREMENT

K-I, G1-I, G2-I

17. Learner can distinguish between pint and quart and recognize a pound in a familiar object like a pound of butter or oleo.

Readiness: Learner knows such terms as heavy, light, bigger than, smaller than and that he can find out how heavy he is by standing on a scale.

- 17a. Measure sand in pints and quarts-- two pints and one quart
- b. Use of scales for weighing children, apples, blocks, sand, clay.
  - c. Use pound boxes to recognize size of a pound--oleo boxes or butter boxes.
  - d. Consider size of milk cartons from cafeteria. Use sand or water to experiment pouring into pints and quarts.
  - e. Using a balance scale and a box of weighing materials (sand, nails, beans, pebbles, etc.) children can answer such questions as: Which is heavier--a cup of beans or a cup of sand?
  - f. For evaluation ask child to demonstrate the fact that sand is heavier than beans and that two pints equal one quart.

# MEASUREMENT -- WEIGHT

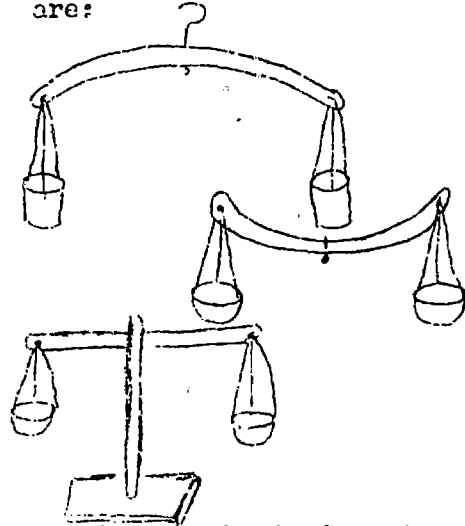
## IV. MEASUREMENT

G3-I, G4-S, G5-R, G6-R

18. Learner can weigh objects to determine which of two is heavier, use the scale to measure pounds and ounces, and solve problems with the ton, ounce, and pound to 85% efficiency.

Readiness: Learner is familiar with weight and can tell which is heavier or lighter of two objects. He also has some familiarity with pint, quart, and pound.

- 18a. Begin with balances by discussing the see-saw, or balance when walking along a wall. Build a balancing device for use in the classroom. Continue with comparing weights of objects until pupils are able to use the word "balance" with understanding. Examples of balances are:



Continue by beginning with something familiar, such as a pound of candy as a standard of measure. Then relate this to measuring a pound of sand, wheat, etc. Class discussions might lead into a list of items purchased by the ounce, by the pound, and by the ton.

- b. Addison-Wesley
  - Book 3-p. 100
  - Book 4-p. 277
  - Book 5-pp. 236-237
  - Book 6-pp. 262-263
- c. See activities 1 and 18
- d. Evaluation
  - $3 \text{ lb} = \underline{\hspace{1cm}} \text{ oz}$
  - $8000 \text{ lb} = \underline{\hspace{1cm}} \text{ T}$
  - $35 \text{ oz} = \underline{\hspace{1cm}} \text{ lb } \underline{\hspace{1cm}} \text{ oz}$

MEASUREMENT - TEMPERATURE

IV. MEASUREMENT

G3-I, G4-S

19. Learner can find freezing, zero, below zero, etc. on a simple, perhaps toy or handmade thermometer with reasonable efficiency.

Readiness: Learner is aware that a thermometer is used to measure his temperature and that how cold or warm the day is, is described by the word temperature.

- 19a. See activity #9 (child made thermometers)
- b. Activities #10, 11, 15
  - c. Addison-Wesley:  
Book 3-p. 94  
Book 4-p. 203
  - d. A big help! Scott Foresman, Teacher's Edition-Book 4-pp. 30-33
  - e. Let various pupils tell about the hottest and coldest weather they have ever experienced. Draw a temperature scale on chalkboard and relate to number line. Point out zero as dividing point.
  - f. Ask various pupils to keep a log for one week of the highs and lows in temperatures of Sheridan recorded by the weather bureau. Some children could do this for other cities and temperatures could be compared.
  - g. Recommended List - #3 and 4
  - h. Suggested reading for teacher help--  
"Arithmetic Teacher," Oct. 1968, pp. 556-559
  - i. Evaluation - Teacher observation may be used to determine understanding pupil's learning.

MEASUREMENT - TEMPERATURE

IV. MEASUREMENT

G5-R, G6-R

20. The learner can solve simple problems involving degrees above and/or below zero on the Fahrenheit thermometer with 85% accuracy.

Readiness: Learner can read a Fahrenheit thermometer and understands that a thermometer indicates temperature.

- 20a. Addison-Wesley  
Book 5-p. 253  
Book 6-p. 59
- b. See activity #1
  - c. See activities #9, 10, 11, 15
  - d. Suggested reading for teacher help: "Arithmetic Teacher," Oct. 1968, pp. 556-559
  - e. Recommended List - #3 and 4
  - f. Evaluation (possible sample)  
Convert each Fahrenheit temperature to Centigrade  
(a) 50° (b) 92° (c) 77° (d) 167°

MEASUREMENT - TIME TELLING

IV. MEASUREMENT

K-I, G1-I, G2-S  
G3-R, G4-R

21. Learner can tell from the clock face the time (1) to the hour (2) the half hour (3) minutes past the hour in terms of 5 minutes or multiples of 5 to 85% efficiency.

Readiness: Learner is aware that clocks and watches "tell time."

- 21a. Addison-Wesley  
Book 1-pp. 195-198  
Book 2-pp. 133-136
- b. See activities #4-7
  - c. See activities #14, 16, 17
  - d. Recommended List - #10 and 11
  - e. Evaluation - See SRA materials for examples of testing devices.

## MEASUREMENT - TIMETELLING

### IV. MEASUREMENT

K-I, G1-I, G2-S  
G3-R, G4-R

22. Learner can solve problems of addition, subtraction and multiplication in clock arithmetic. This work is optional and for enrichment.

22a. Addison-Wesley-Book 4-pp. 230-235  
b. For evaluation the following could be used:

- (1) 2 hrs after 9 o'clock is \_\_\_\_  
o'clock--- $9 + 2 =$  \_\_\_\_
- (2) 8 hrs after 6 o'clock is \_\_\_\_  
o'clock--- $8 + 6 =$  \_\_\_\_
- (3) 2 hrs before 3 o'clock is \_\_\_\_  
o'clock--- $3 - 2 =$  \_\_\_\_
- (4) 4 hrs before 3 o'clock is \_\_\_\_  
o'clock--- $3 - 4 =$  \_\_\_\_
- (5) 5 hrs before 8 o'clock is \_\_\_\_  
o'clock--- $8 - 5 =$  \_\_\_\_

## MEASUREMENT - CALENDAR

### IV. MEASUREMENT

K-I, G1-I, G2-I

23. Learner can name the days of the week and intuitively learns the months of the year through constant discussion of the calendar.

Readiness: Learner is aware of time through usage of the vocabulary: Today, yesterday, tomorrow, this week, etc.

23a. The days of the week become important when we begin relating to Monday as the first day of school in the week. Sunday is church day for most people. Friday is the last school day of the week, etc.

The important months might be the month in which the child was born, Christmas in December, September when school begins, etc.

- b. See activities #19-22
- c. For evaluation, activity #19 may be used effectively through constant use with teacher observation.



# MEASUREMENT - CALENDAR

## IV. MEASUREMENT

G3-S, G4-R, G5-R, G5-R

24. The learner will be able to tell with 85% efficiency: the number of seconds in a minute, minutes in an hour, hours in a day, days in a week, weeks in a month and year, months in a year.

Readiness: The learner is aware that the calendar shows days. In previous grades, days of the week and holidays are located and the month named. This is done as a daily routine but mastery is not stressed.

- 24a. Addison-Wesley-Book 3-pp. 217, 280  
 b. Activity #8 could be used for grades 3 and 4  
 c. Recommended List #3, two filmstrips on the calendar  
 d. Evaluation

Complete the following:

208 wk =      yr  
 4 hr =      min  
     da = 72 hr  
     sec = 3 min  
 56 da =      wk  
 5 yr =      mo  
 4 yr =      da

25. Learner can gain an intuitive understanding of decade, century, millennium.

- 25a. Addison-Wesley-Book 4-pp. 34-35

## ACTIVITIES - MEASUREMENT

1. The following is a suggested list of materials from Arithmetic Teacher, December 1967, p. 652.

<u>Item</u>	<u>Source</u>	<u>Cost</u>
Clocks	Old clock from home	\$0
Thermometer	Supermarket or hardware	Under \$1.00
Kitchen or baby scales	Used furniture store, Salvation Army, Goodwill	Under \$1.00
Cubic-inch blocks	School Supply Co.,	Under \$3.00
Cubic foot	R.H. Stone Products, Box 414, Detroit	Under \$5.00
Square foot	Cut cardboard	\$0
Square inches	Tile store	Under \$1.00
Milk Containers	Home	\$0
Egg Cartons	Home	\$0
Gallon Jug	Child donation	\$0
Weights (or sand etc)	Various	\$0
Baby bottle, with ounce graduations	Child donation	\$0
Kitchen measuring cup	Supermarket	\$.25
Coffee Cup	School kitchen	\$0
Eye dropper	Medicine bottle	\$0
Funnel	Supermarket	\$0
Teaspoon	Kitchen	\$0
Tablespoon	Kitchen	\$0
Ruler	Teacher	\$0
Yardstick	Lumber company	\$0
Tape measure	Hardware	\$1

## 2. Helping Mother 1-2

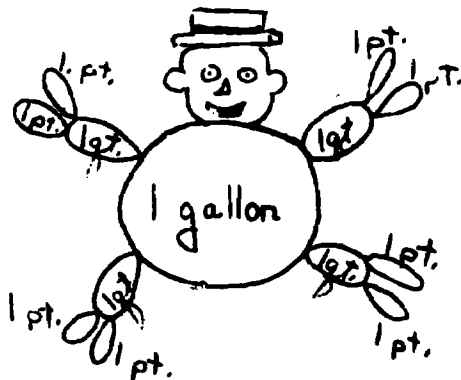
Purpose: Recognizing  $1/2$  pint, pint, quart, 2 quart

Materials: Empty milk cartons-- $1/2$  pint, pint, quart, 2 quart

Directions: You are about to bring in the milk the milkman has just left at your house. Alex will be the milkman and leave some milk at someone's house. That child will bring the milk in, put it in the refrigerator (table) and tell the class what he brought in. For example: I brought in 2 quarts of milk and  $1/2$  pint of cream. Continue using different milkmen,

## 3. Mr. Measure Man 1-3

When teaching liquid measure, use a figure called "Mr. Measure Man." He can be made out of bristol board with a red hat, green hands and feet, yellow legs and arms, white face, and black body. Useful in helping children understand relationship between pints, quarts, and gallons.



## 4. Paper Plate Clocks 1-2

Each child can make his own clock from a paper plate with large numerals cut from a calendar and pasted around the rim. Use two strips of Oaktag for hands and a paper fastener to hold them together.

## 5. An Unusual Clock - 2

The second grade children in one class found the problem of learning to tell time much easier and more fun with the aid of a large table-sized clock. This clock was originally a round dining room table, but with the use of a saw and a paintbrush it became a worthwhile learning tool. It's height is 23 inches from the floor. The large black numerals are easily read even at a distance. The moveable hands are bolted on through a hole in the middle, and can be removed when the table is to be used for other purposes.

### 6. Human Clock 1-2

**Materials:** Use chalk to draw a very large clock face on the schoolroom floor. Cut the hands. Take two ribbon streamers. Have a child sit in the center of the clock to hold one end of each ribbon. Let one ribbon (the minute hand) extend to the rim of the clock. Make the other ribbon (the hour hand) noticeably shorter.

**Directions:** Today you are going to make a Human Clock. There is a large clock face drawn on the floor. I am going to choose two helpers to hold ribbon, which will be the hands of the clock. One we will call Mr. Hour, the other Miss Minute. Mr. Hour will hold the short ribbon and walk around to make the clock say the different hours that I name. Miss Minute may hold her long ribbon at twelve until she learns the work of Mr. Hour. Then she will move to show the minutes. (Let each pair of children illustrate several examples) This is very good for slow learners.

### 7. What Time Is It? 1-3

**Materials:** Hand made paper plate clocks. Divide the class in two teams.

**Directions:** I will show this clock to the first person on team A. If he can tell me what time the clock shows, he will win a point for his team. I will reset the clock and show it to the first person on team B and so on. I will keep track of your scores. At the end of the playing time, the team with the highest score will be the winner.

### 8. Our Own Calendar - 2

Children may construct their own calendar as an aid to understanding how days and months are measured. Take twelve 12" x 18" sheets of construction paper, ruling the lower part of each sheet in 1 1/2" squares. Help children fill in squares with the correct numbers for each month and to note that some squares are left blank. Children also learn the names of the days of the week and may record important holidays. They may even illustrate the holiday such as a turkey for Thanksgiving.

### 9. Making a Thermometer - 4

The abler student may devise a large thermometer in which the tube of liquid is represented by a movable string. They will need a piece of cardboard 5" x 18". They should draw the outline of a tube and copy the calibration marks and numerals from a thermometer. The scale should range from approximately 20 degrees below zero to 130 degrees above. Holes should be made at both top and bottom of their tube illustration so the string can be inserted. One half a piece of heavy white string should be colored red-- the string being about 32 inches long. The white half of the string should then be inserted through the top hole from the front, and the red half inserted through the bottom hole from the front. The two halves should be tied at the back of the thermometer just tightly enough so that the newly formed band of string can be rotated.

10. Place a weather thermometer outside where pupils can easily read it. Ask them to read the temperature at lunch time and again before they go home for the day. A record could be kept or a bar graph made.

11. Have various kinds of thermometers brought to school--cooking, fever, weather, etc. Introduce the name Fahrenheit. Explain how the colored liquid in the tube works. When the air around the thermometer grows warmer, the liquid grows warmer, expands and fills more space inside the tube. When the air grows cooler, the liquid contracts and goes down to take up less space.

## 12. Equivalents

Make a set of cards containing on front and back such information as follows:

<u>3 ft = ? in</u>	<u>(36)</u>	<u>144 sq in = ? sq ft</u>	<u>(1)</u>
front	back	front	back

Put cards on the table face up and have pairs of children compete by taking turns to supply the missing number. Cards incorrectly answered can be put in separate pile for study. An individual could also use these for studying.

## 13. Two Deep

Form two circles as for the outdoor game of the same name. The object is to keep one's place in the inner circle. The teacher or a pupil flashes a measurement equivalent card: 1 foot = ? in, etc. and nods to some child in the inner circle. He must give the correct answer before his partner in the outer circle or change places with him. Caution should be used in partner selection so that the partners will be reasonably equal in arithmetic ability.

## 14. Tick-Tock

Togged out with cardboard or pie plates, each with a number from 1 to 12, twelve children stand in a circle to form a clock. All face the same direction. A thirteenth child, the cuckoo, goes to the center of the clock face holding two dangling strings, one black to represent the hour hand and one red for the minute hand. The leader calls "five o'clock." "5" dashes to the center to grab the black "hour hand" string, while "12" races for the red "minute hand" string. They run back with the strings to their positions in the circle. When the player in the center feels the strings are taut, he calls out "Cuckoo!" The game will get harder when the leader calls a number that the children have to puzzle out, such as "five fifty-five or five to six." Then six and eleven must dash for the strings. Still later you might try a few time problems, such as this one: It takes 20 minutes for Jane to get to school. She leaves home at eight o'clock. What time does she get to school?

### 15. Zipper Thermometers

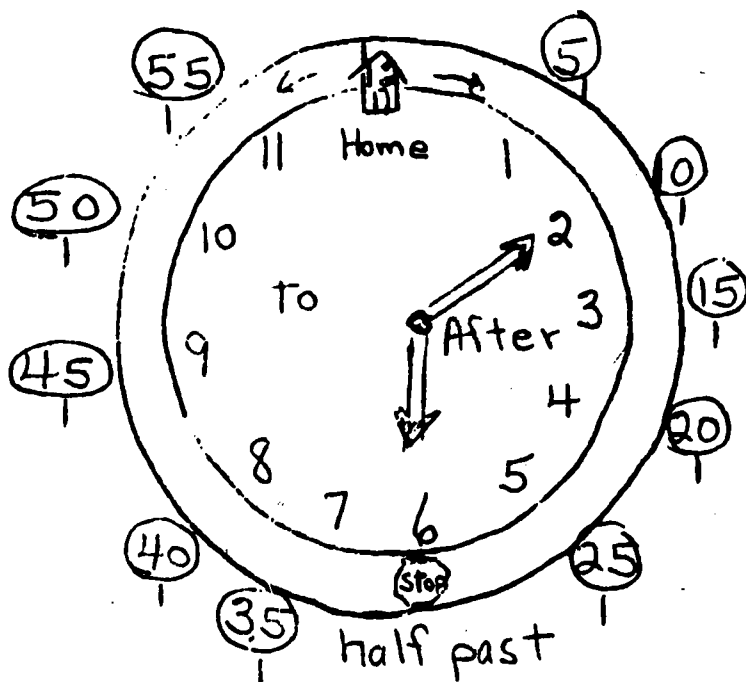
This is good for teaching negative and positive numbers and makes it more meaningful and easy to read. Slit a piece of cardboard and sew or staple an 18" zipper in the slit. Make appropriate markings beside the zipper on the cardboard. Paint the tape of the zipper on the cardboard, with bright red nail polish so it stands out. Each morning, let one child take the outdoor reading from a thermometer on the outside of the window. Then adjust the zipper. Additional thermometer reading can also be taken during the day if a marked change can be asked to tell the difference in the number of degrees.

### 16. Time Telling

Around two permanently painted circles on the gym floor, children learn to place evenly the numerals from 1 through 12. Two teams of partners play "What Time Is It?" When leader calls, "Five minutes to three," one partner steps in front of 3. The other steps outside the circle opposite 11. Partners in place first score for their team.

### 17. Time Telling

The circumference of a clock drawn on heavy paper or cardboard represents a road. Twelve o'clock is "home" and is marked by a sketch of a house. A stop sign is located at six o'clock. Distance markers indicate MINUTE MILES between the hours. Count by fives to answer: "How far is it from home to stop sign?" "How far from stop sign to home?" "How far from home all around the road back to home?"



18. Let each pupil use a bathroom scale to weigh himself and weigh another object. Then subtract to find the weight of the object. The pupils could also weigh individual objects, then weigh them together to see if the combined weights will equal the sums of the individual weights.

19. For opening exercises, name the month, the date, the year, and the day of the week. You might like to set this up on a rotation basis so a new pupil is selected each day for this naming process.

20. The Birthday Train

This consists of an engine with twelve cars representing each month of the year. Pictures are taken of all children and placed on the car of their birth month. Follow up with periodic discussions of the train in relation to the monthly birthdays.

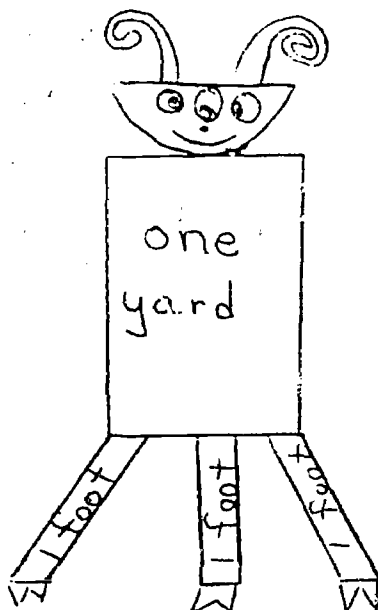
21. Ask the pupils to locate birth days of important people and dates of important events. For example: February--Washington's birthday, Lincoln's birthday, Beethoven's birthday, etc.

22. Make a large construction paper birthday cake. Make a candle for each child, including the month, his picture, and the date of birth. On the child's birthday, place a large flame upon his candle.

23. Martian Measure Man 2-3

Materials: Make "Mr. Three M" of construction paper. Body = 1 yard long, Feet = 1 foot long, Toes = 1 inch long. Different colors of paper could be used for different measurements. Width and head could be the size you choose.

Directions: Children could design their own Martian Man but must use yardstick and ruler. By making their own they see the relationship of common linear measurement.



## 24. Lengthy Problem

One person in the class will be the leader. While class members put their heads on the desks with eyes closed, the leader measures an object in the room. He then tells the class the length of the object. Pupils will then need to guess what the object measured might be and prove it by measuring the object. The first person to guess and measure the correct object will be the leader for the next round.

## 25. Midnight

Materials: two packs of cards (on each card is a clock face showing a certain time and below it the time is written out) No two cards are alike. The second pack is like the first except the time is not written on each card. Markers for children,

Directions: Cards without time written on them are dealt, face up. Each player has the same number of cards (about 8 is good). When everyone is ready the caller who has cards with time shown and written holds his cards and reads them one at a time. Each player places a marker on the clock which shows the time called, if he has it. The first player to cover all his cards calls MIDNIGHT. The caller then checks the player to see if he is correct by having him read the time on each clock back to him. As the player reads, the caller must find the corresponding card among the cards called during the game.

## 26. Research Fun

Here is a list of measurement terms, some of which are now outmoded. They are interesting however, and might provide a good activity for a fast learner (or a group). After researching these terms a child might prepare a bulletin board for the class.

pottle

gill

gaggle

fathom

cubit

hand

palm

span

digit

yard

journal

inch (3 barley corns)

link

chain

noggin

furlong

bolt

ell

pole

vara

hogshead

skein

cable

hank

league

decade

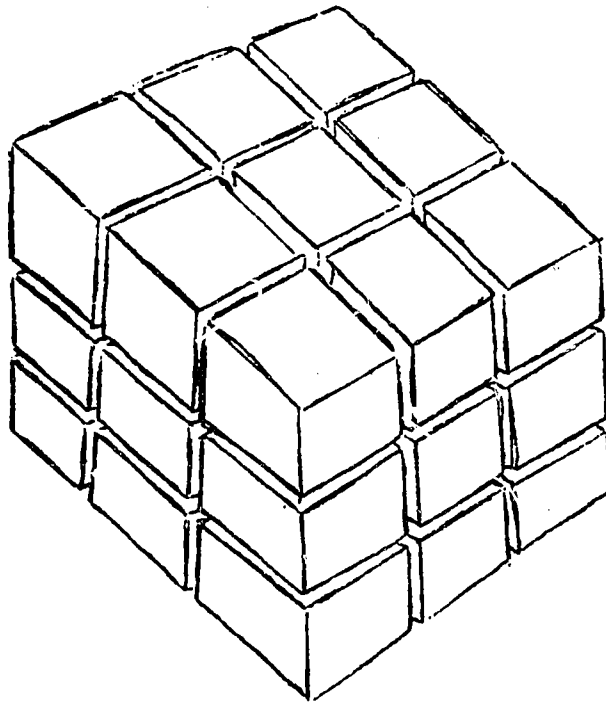
millenium



27. Hector and the Moldy Cheese

Hector's mother gave him a cube of moldy cheese. All sides were covered with mold. Hector cut the cheese into 27 little cubes as you see below.

- A. How many of Hector's little cubes of cheese had no mold?
- B. How many had just one moldy side?
- C. How many had two moldy sides?
- D. How many had three moldy sides?



## MEASUREMENT - RECOMMENDED PURCHASE LIST

### TRANSPARENCIES

Instructo Products Co.  
Philadelphia, Penn. 19131

389 Dry Measures  
389 Linear Measures  
389 Liquid Measures  
389 Weight

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### FILMSTRIPS

Eye Gato Filmstrips

133H Meaning and Use of Fractions and Measures  
162G Measures  
162H Perimeters and Areas  
- - - - -

McGraw Hill filmstrips

How a Day Passes  
Busy Week  
Month  
Year  
History of Area  
History of Calendar  
History of Time  
History of Linear Measurement  
History of Weight and Volume  
History of the Number System  
- - - - -

Society for Visual Education, Inc.

537-21 Calendar, Time and Temperature  
532-20 Space Geometry, Surface and Volumes of Space Figures  
532-15 Using Measure: - Time, Temperature, Linear, Liquid, Dry

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### DITTO MATERIALS

Jenn Publications  
815-825 E. Market St.  
Louisville, Ky. 46206

1-150 Thermometer - Gr. 1	B-334 Linear - Gr. 2	B-337 Ruler - Gr. 2
1-151 Clock - Gr. 1	B-335 Ruler - Gr. 2	B-338 Thermometer G
B-333 Linear - Gr. 2	B-336 Ruler - Gr. 2	B-341 - 343 Clock

## MEASURING DEVICES

Ideal School Supply Co.  
11000 So. Lavergne Ave.  
Oak Lawn, Illinois 60453

Metric Place Value Chart - #777 - \$2.50  
Perimeter Area Board - #763 - \$3.25  
Meter Stick - #730 - \$.50  
One Cubic Yard - #776 - \$7.50 Grades 5-8  
Cubic Foot - #761 - \$2.20 Grades 5-8

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## RECORDS

Catalog, Learning Arts  
P.O. Box 917  
Wichita, Kansas 67201

Learn to Tell Time With Grandson Clock - \$1.89 Interesting for primary grades

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## GAMES

Milton Bradley  
Springfield, Mass.

Tell-Time Quizmo

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## A NEW AND EXCITING AID!

General Electric,

Radio, Phono, Viewer - looks like a T.V. set. Synchronizes a record and film strip. Records are titled Show'n Tell. Those especially suited to measurement are (1) Time, (2) Days, Weeks, and Seasons of the Year, (3) Clocks Record  
Price: about \$1.00 (includes the tape) Viewer about \$35.00.

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## ADDITIONAL MATERIALS

Addison-Wesley -- Duplicator Masters (Robert Eicholz and Phares O'Daffer)

Harcourt Brace -- Learning to Compute, 2nd edition, for inventory practice and evaluation

Merrill Math Series -- Teacher's Edition

RECOGNITION OF MONEY

V. - MONEY

G1-S

G2-S, E

G3-6-R

1. The learner will recognize and differentiate between pennies, nickels and dimes and also be able to recognize the cent symbol (¢) by counting play or real money or by circling the correct amount shown on a written page with 85% accuracy.

Readiness: Counting skills.

- 1a. Addison-Wesley-Grade 1-pp. 33-34, 123-124, 179-180.
- b. Addison-Wesley-Grade 2-p. 121.
- c. No. 1501 Playstore cash register 7"x7"x7" by Ideals
- d. No's 5601-5504 Toy money from Ideals
- e. Real Money
- f. See activities #2, 3, 4
- g. Refer to materials page

2. The learner will recognize and differentiate between quarters, half-dollars and dollars and recognize the symbol for dollars (\$) by counting play or real money or by circling the correct amount shown on a written page with 85% accuracy.

Readiness: Counting skills.

- 2a. Addison-Wesley-Grade 2-pp. 271-272
- b. Addison-Wesley-Grade 3-p. 108
- c. Jenn Publications-Grade 2-No's B351 through B359
- d. Jenn Publications-Grade 3-No's C97, C169 through C174
- e. See activities #1-14
- f. Refer to materials page

MONEY OPERATIONS

V. MONEY

G1-S

G2-6-E

3. The learner will demonstrate an understanding of money value by solving problems dealing with money with 85% accuracy.

Readiness: Computation skills.

- 3a. Addison-Wesley-Grade 1-pp. 193-194, 219-220, 265-266, 273-276  
b. Addison-Wesley-Grade 2-pp. 131-132, 273-274  
c. Addison-Wesley-Grade 3-pp. 78, 80, 119, 204  
d. Addison-Wesley-Grades 4-6, Widely scattered throughout the book  
e. No. 764 Dollar and Cents Place Value Kit-Grade 3-4 from Ideal  
f. Refer to activities #1-14  
g. Refer to materials page

## ACTIVITIES - MONEY

### 1. Going Shopping - Grade 2

Materials: coins (quarters, nickels, pennies, dimes)

Directions: "I'm going shopping with a quarter and a dime." "I'm going to buy a book that costs 40¢. Have I enough money?" When a child answers "no" he must be able to tell what is needed. The child who is correct gets to go shopping next.

### 2. Who Will Trade? - Grade 1-2

The child who is "It" offers to trade a set of coins. He says for example, "I have two nickels and one penny. Who will trade with me?" He calls on a child who raised his hand. The child may offer him one dime. If "It" refuses, he may have another turn. If he accepts, he loses his turn because he has accepted coins of less value. The children love to "catch" each other.

### 3. I Have - Grade 1-2

Let a child pick up some coins secretly. To the group he will say "I have three coins and they have the same value as 15 pennies. What coins do I have?" The child who answers correctly gets the next turn.

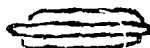
### 4. Cafeteria Fun

Materials: Children need pencils and paper. On the board draw items and their prices of things that could be bought in a cafeteria. Toy cash register, coins.

Examples:



Pie 10¢

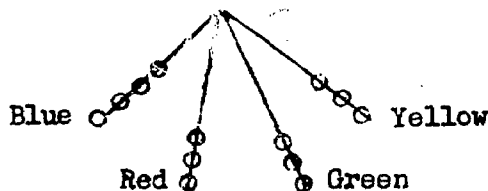


Hotdog 15¢

Directions: Today you may take some of your friends to lunch. They will choose what they want and you will pay for it. You will need to write your friends name and what he wants to eat on a piece of paper. You will have to add up each persons food and pay for it.

### 5. How Much Does the QUIPU Show? 3-4-5

Ancient Incas of Peru used a counting device called a quipu. The quipu was made of cords of various colors, each with a special meaning, with knots to record quantities. The quipu shown records the amount of money a woman owed the grocer. The blue cord shows dollars, the red cord shows dimes, the green cord shows nickels and the yellow cord shows pennies. How much does the woman owe? \$4.48



## 6. Number Line

To teach equivalent values, a number line on the chalkboard or a chart is helpful.

0	5	10		
1				
2	5			
3				
4	5			
5			25	
6	5	10		
7				
8	5			
9				
10	5	10		50
11				
12	5			
13			25	
14	5			
15		10		
16	5			

- a. As children work with the number line bring out these equal values:

5 pennies = 1 nickel  
 10 pennies = 1 dime  
 2 nickels = 1 dime  
 5 nickels = 1 quarter  
 2 dimes + 1 nickel = 1 quarter  
 2 quarters = 1 half dollar

- b. Extend to one dollar  
 c. Use the number line to determine different ways of making up given amounts of money: 25¢, 35¢, 70¢.  
 d. Use the number line to help children determine the coins needed in change.

(American Book Co., 1966 Book 3)

## 7. Flannelboard to Demonstrate the Equivalence of Money - Grade 3-4

Set up drawings or cutouts to show:

Nickel 5¢	Penny 1¢
○	○ ○ ○ ○ ○
○	○ ○ ○ ○ ○
○	○ ○ ○ ○ ○
○	○ ○ ○ ○ ○

1 nickel is worth 5 pennies  
2 nickels are worth 10 pennies  
3 nickels are worth 15 pennies  
4 nickels are worth 20 pennies  
and so on

Dime 10¢	Nickel 5¢
○	○ ○
○	○ ○
○	○ ○
○	○ ○

1 dime is worth 2 nickels  
2 dimes are worth 4 nickels  
3 dimes are worth 6 nickels  
4 dimes are worth 8 nickels  
and so on

Dime 10¢	Penny 1¢
○	○ ○ ○ ○ ○ ○ ○ ○ ○ ○
○	○ ○ ○ ○ ○ ○ ○ ○ ○ ○
○	○ ○ ○ ○ ○ ○ ○ ○ ○ ○
○	○ ○ ○ ○ ○ ○ ○ ○ ○ ○

1 dime is worth 10 pennies  
2 dimes are worth 20 pennies  
3 dimes are worth 30 pennies  
4 dimes are worth 40 pennies  
and so on

8. Refer to activity #24 - multiplication-division section.

9. How Many Bills? 3-4-5

An interesting item for the bulletin board:

A man has \$80 in paper money in his pocket. He has the same number of \$1, \$2, and \$5 bills. How many bills of each kind does he have? Answer - 10



10. How much did Jack Make? 3-4-5

Jack sold a water pistol to John for 30¢. John sold it back to Jack for 20¢. Jack sold it again for 35¢. How much did Jack make?--Answer: 15¢--The original sale price of 30¢ was the value. Jack bought the pistol back for 10¢ less than its value and sold it the second time for 5¢ more than its value.  $10¢ + 5¢ = 15¢$

11. Selecting the Right Coins Quickly

1. Which of these amounts could be paid with three different coins?  
a. 30¢ b. 36¢ c. 52¢ d. 60¢
2. Which of these amounts could be paid with four different coins?  
a. 37¢ b. 51¢ c. 81¢ d. 75¢
3. Which of these amounts can be paid with three different coins?  
a. 16¢ b. 21¢ c. 26¢ d. 35¢
4. Which of these amounts can be paid with four different coins?  
a. 16¢ b. 31¢ c. 44¢ d. 66¢
5. Which of these amounts can be paid with three different coins?  
a. 45¢ b. 56¢ c. 75¢ d. 86¢
6. Which of these amounts can be paid with four different coins?  
a. 81¢ b. 90¢ c. 96¢ d. 98¢
7. Which of these amounts can be paid with three different coins?  
a. 75¢ b. 85¢ c. 90¢ d. 95¢
8. Which of these amounts can be paid with four different coins?  
a. 90¢ b. 95¢ c. \$1.05 d. \$1.06

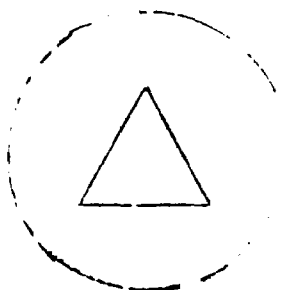
12. What's Missing? 5-6

For a quick quiz some day, duplicate some problems. In each of them leave out a part necessary in order to solve the problem. For example, "Mrs. Jones bought a dozen eggs. If she gave the clerk \$1.00, what change did she get?" Children will need to find what part is missing (price per dozen), put in a reasonable figure for the missing part (\$.70), and then solve the problem. Another example might be: "Bobby got a sweater at a sale for 20% off. What did he pay for the sweater?"

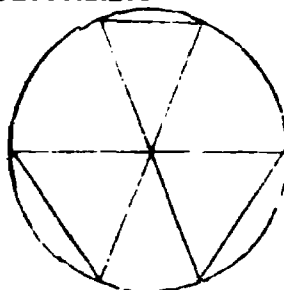
### 13. Globonia - Grade 4

In a land called Globonia there was a king who could only count to three so the money of Globonia had to be based on three. The coins of Globonia were:

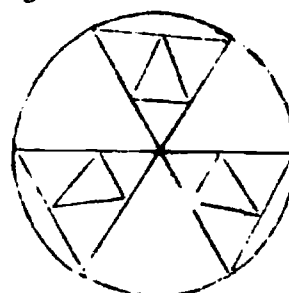
The globenny was their coin of smallest value.



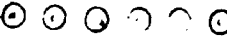


The coin of next greatest value was the globnic which was worth three Globennies



The third coin was the Globona which was worth three globnics.



1.  Here are three globennies. They are worth (1) globnic and 0 globennies.
2.  Here are five globennies. They are worth (1) globnic and (2) globennies.
3.  Here are six globennies. They are worth (2) globnics and (0) globennies.
4. People in Globonia used only three digits, 0, 1, 2, to name amounts of money. The chart shows how they used the digits. Finish it.

	<u>( globnics</u>	<u>lobennies</u>
one globenny		1
two globennies		2
three globennies or one globnic	1	0
one globnic one globenny	1	? (1)
one globnic two globennies	? (1)	2
two globnics	2	? (0)
two globnics one globenny	? (2)	? (1)
two globnics two globennies	? (2)	? (2)

5. In Globonia the numeral 10 is read "one-zero." It means 1 globnic and 0 globennies. The numeral 12 is read "one-two." It means 1 globnic and (2) globennies.
6. In Globonia the numeral 21 means (2) globnics and (1) globenny.

7. In Globonia the numeral 20 means (2) globnics and (0) globennies.
8. In Globonia the numeral 22 means (2) globnics and (2) globennies.
9. In Globonia the numeral 11 means (1) globnic and (1) globenny.

### Working with the Globona

1. A globona was worth three globnics. How many globennies was it worth? (9)
2. One globona and one globnic are worth (12) globennies.
3. How many globennies are 2 globonas worth? (18)
4. Nine globennies are worth (3) globnics. They are also worth (1) globonas.
5. Fifteen globennies are worth (1) globona and (6) globennies.
6. Suppose you were in Globonia and a child wanted to trade you a globona for a globnic and two globennies. Would you trade? (yes)  
Why? (you would receive the value of 9 globennies for 5 globennies)
7. Suppose a child in Globonia wanted to trade twelve globennies for a globona and a globnic. Would it be a fair trade? (yes) Why?  
(1 globona = 9 globennies, 1 globnic = 3 globennies  $9 + 3 = 12$  globennies)
8. Complete the chart below.

	Globonas	Globnics	Globennies
one globona	1	0	0
one globona one globenny	1	0	? (1)
one globona one globnic	1	? (1)	? (0)
one globona two globnics	? (1)	? (2)	? (0)
one globona two globnics one globenny	? (1)	? (2)	? (1)
two globonas one globnic 2 globennies	? (2)	? (1)	? (2)
two globonas two globnics two globennies	? (2)	? (2)	? (2)

9. Licorice sticks in Globonia cost two globennies each. How many could you buy for a globona? (4) What change would you get?  
(1 globenny)
10. Apples cost one globnic and one globenny each. How many could you buy for a globona? (2) What change would you get? (1 globenny)
11. If you bought a baseball card for two globnics and sold it for seven globennies, would you make a profit? (yes) How much profit?  
(1 globenny)
12. You want to buy some crayons that cost two globonas. You have three globnics and seven globennies. You need (2) globennies more.

14. Money chart to use for teaching "making change"

Materials: one 24" x 24" cardboard  
one 2" x 24" for pocket  
one 1/2" x 24" for pocket  
two 1" x 1 1/2" for pockets  
many 5" x 22" for making problems  
play or real money

example:

making change			
you spend	give clerk	count change	you received
18¢	25¢	18....	17¢

Directions: Place problem strip on the chart. Pupil counts change silently. Pupils place coins on desks for amount of change needed. Have one child demonstrate next problem.

## MONEY

### FILMSTRIPS

Society for Visual Education Inc.  
1345 Diversey Parkway  
Chicago, Illinois 60614

531-5 Money--Penny, Nickel, Dime, Quarter

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### FILMS

University of South Dakota  
Film Library  
Extension Division  
Vermillion, South Dakota

"Arithmetic is Easy"--11 minutes  
"Arithmetic in the Food Store"--11 minutes  
"Making Change for a Dollar"--11 minutes

Intermediate

"The Story of Our Money System"--11 minutes

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### TRANSPARENCIES

Colburns  
2702 Montana Ave.  
Billings, Montana 59101

Mathematics Readiness Series 1400--Includes transparencies on money

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### RECORD

American Encyclopedia of Learning through Music  
"All about Money"

## IDENTIFYING AND NAMING COMMON PLANE FIGURES

### VI. GEOMETRY

K-2-I G3-S

1. Given models of circles, squares, rectangles, and triangles (wire, paper or flannel cutouts, pencil or chalk outlines), the student can identify, name orally, and distinguish among the plane geometric figures.  
  
Readiness: Teacher observation
- 1a. With the use of a pegboard and rubber bands, the student can construct a square, rectangle, and triangle.
- b. The student with the use of pencil or chalk can make drawings of circles, rectangles, and triangles.
- c. See activities # 1-4, 10, 11, 14, 15, 17, 18, 22-28
2. Given models of line segments of different lengths, the student can identify the longest and shortest.
- 2a. The student can further his understanding of the concept length by drawing a line segment of given length (whole units).
- b. Let the student construct a line segment of a given length. For example: Draw a line 4 inches long.
- c. See activity # 25
3. Using a straightedge the student can draw a recognizable square, rectangle, and triangle.
- 3a. See activity # 26

# IDENTIFYING AND NAMING COMMON PLANE FIGURES

VI. GEOMETRY

G3-I G4-S G5-R

4. Using a ruler, the student can construct a number line and label the points with whole numbers.

4a. See activity # 15.

# IDENTIFYING AND NAMING SPACE FIGURES

VI. GEOMETRY

G3-I G4-S G5-R

5. Given a verbal description of a precise location in the classroom, the student can locate and identify the point. Example: Where is the place the floor and those two walls meet?

Properties: Length, simple closed plane figures, and area.  
SEE MEASUREMENT

Readiness: To be able to identify and name common plane figures.

5a. Give the students verbal descriptions of precise locations in the classroom, and have them locate and identify these points. Example: Where is the place where the two walls and the ceiling meet?

6. The student can identify, name, and distinguish among these space model figures (cubes, spheres, cylinders, and cones). These can be wood or plastic solids, rolled paper, etc.

6a. When given models of space and plane geometric figures, the students practice naming, identifying, and distinguishing among them.

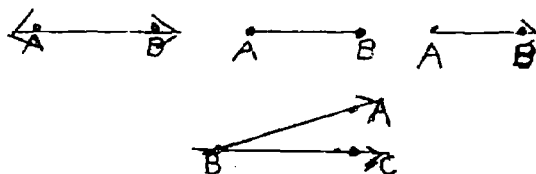
# IDENTIFYING AND NAMING COMMONSPACE FIGURES

## VI. GEOMETRY

G3-1 G4-S G5-R

7. Using a straightedge and folded paper, the student can construct a right angle.

- 7a. Students can make rough pencil and/or chalk drawings (outlines) of the plane geometric figures.
- b. Using a straightedge, the students can construct models for lines, line segments, rays, and angles and label them in the following ways;



8. The learner can construct a circle when given the center, the radius and a compass to use.

8. The learner can be given a set of points to draw satisfying certain conditions which are given to him.

Example:

The set of all points one inch from a given point is a (circle)

The set of all points contained in two rays with a common end-point is an (angle).



IDENTIFYING AND NAMING COMMON SPACE FIGURES

VI. GEOMETRY

G4-I G5-S G6-R

- |   |   |
|---|---|
| <p>9. Using models including a triangle, right triangle, quadrilateral, parallelogram, square, and rectangle, the student can identify, name, and distinguish among them.</p>                       | <p>9a. From a group of models of plane figures (wire, paper or flannel cut-outs, pencil or chalk outlines) the student can identify, name, and distinguish among them. (Models can include; curves, line segments, angles, triangles, parallelograms, squares, rectangles, rhombus, other figures could be pentagon, hexagon, octagon)</p> <p>b. See activities no. 7&amp;8, 11-14, 17, 18, 21, 23-26</p> |
| <p>10. The learner can identify, name, and distinguish among the following space figures when presented with the models of a cube, right rectangular prism, sphere, cylinder, cone and pyramid.</p> | <p>10a. From models of space figures (prism, sphere (hemisphere) cylinder, cone, pyramid) the students can identify, name, and distinguish among them. Models can be wood or plastic solids, paper models, sketches etc.<br/>L.W. Singer, Book 6, pp. 392-409 (1966)</p> <p>b. See activities 21,23 and 26.</p>   |

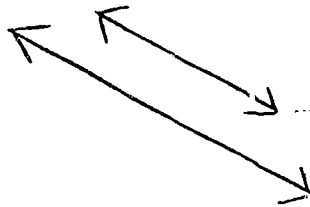
IDENTIFYING AND NAMING COMMON SPACE FIGURES

VL GEOMETRY

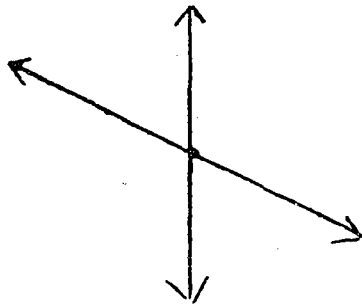
G4-I G5-S G6-R

Perimeter (SEE MEASUREMENT)

11. The student can sketch and describe parallel lines. Example: Parallel lines are lines in the same plane that never meet.



12. The learner can construct and describe intersecting lines.



11. Students can sketch, give descriptions, show examples of parallel lines.

- 12a. Students can sketch, give descriptions, show examples of intersecting lines.

- b. See activities no. 17,25.

IDENTIFYING, NAMING AND CONSTRUCTING PLANE  
AND SPACE FIGURES.

VI GEOMETRY

G5-I G6-S G7-R

13. The student can describe a plane geometric figure as a set of points.

Example:

A circle is the set of all points in a plane a fixed distance from a given point.

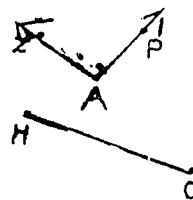
An angle is the set of all points contained in the union of two rays with a common endpoint.

Readiness: To be able to name and identify space figures.

14. The learner can read and write standard notation for plane figures.

- 13a. Give students the centers radii for different circles and let them construct these circles using a string or a compass.  
b. See activity no 25.

- 14a. The student can practice drawing plane figures using standard notations. Examples:



is denoted by  $\angle ZAP$

is denoted by  $\overline{HO}$

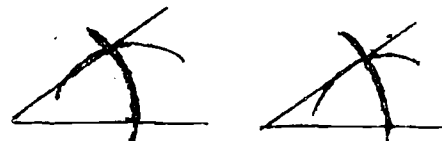
Concept of volume (SEE MEASURE-  
MENT)

IDENTIFYING, NAMING AND CONSTRUCTING PLANE  
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VI GEOMETRY

G5-I G6-S G7-R

15. The learner can give examples of, describe, and sketch perpendicular lines. Examples: Perpendicular lines are intersecting lines that form right angles. The top edge and side edge of the window are perpendicular to each other.
16. When given a pair of line segments, angles, triangles, or other polygons, the student can identify the pairs as congruent or not congruent by matching the figures in some manner. (trace and overlay, cutouts, etc.)
17. The student can construct a plane figure congruent to a given line segment, angle, or triangle with the use of a straightedge and compass.
- 15a. Students can practice sketching perpendicular lines. They can describe them. Examples of things perpendicular can be located in the classroom and outside.
- b. See activities # 17, 25, 26.
- 16a. Have students tell in their own words what they think congruent angles and triangles are. Have them through the use of cutouts, trace and overlay etc. show that two angles are congruent. This can be done to show that two triangles are congruent; can be done with other figures, also.
- b. See activities # 6, 17, 25.
- 17a. Have students practice constructing plane figures that are congruent to given plane figures. Example:



**IDENTIFYING, NAMING, AND CONSTRUCTING PLANE  
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VI. GEOMETRY

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18. The learner can describe a given plane figure as a set of points.

- 18a. Let students practice describing various plane figures as sets of points, this can be written or given orally.  
b. See activities # 25 and 26.

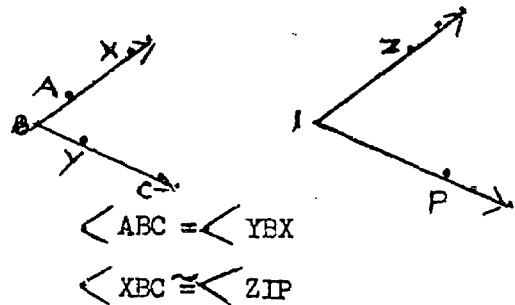
19. The student can describe a given space figure as a set of points.

- 19a. Students can practice describing (written or orally) space figures as sets of points.  
b.

Perimeter of a polygon, area of any parallelogram and triangle, and volumes, SEE MEASUREMENT.

20. The learner can distinguish between equal and congruent figures.

- 20a. When given several equal figures and congruent figures the students can distinguish between the equal and congruent figures. Example:



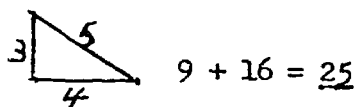
IDENTIFYING, NAMING, AND CONSTRUCTING PLANE  
AND SPACE FIGURES

VI GEOMETRY

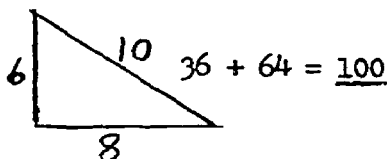
G5-I G6-S G7-R

21. The learner can test the Pythagorean rule with the following triangles whose sides are:

(A) 3; 4; 5.



(B) 6; 8; 10.



22. Using a straightedge and compass the learner can construct the bisector of a given angle.

- 21a. See activity # 16.

- 22a. Have students practice using the compass and straightedge to bisect a group of given angles.  
Example:



23. Using a straightedge and compass the learner can construct a line perpendicular to a given line at a given point.

- 23a. See Addison-Wesley Book 6 p. 199.

IDENTIFYING, NAMING, AND CONSTRUCTING PLANE  
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VI GEOMETRY

G5-I G6-S G7-R

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|--|--|
| 24. Using a straightedge and compass the student can construct the perpendicular bisector of a given line segment.                 | 24a. Have students draw several line segments, then using a compass and straightedge students can bisect them.   |
| 25. Given a line and a point not on the line, the learner can construct a line parallel to the given line through the given point. | 25a. See Addison-Wesley Book 6, (1968) pp 116-119.<br>b. Have the students construct parallel lines. (Give a line and a point not on the line, have students construct a line parallel to the given line through the given point). |
| 26. Using a compass, the student can construct a circle with a given center and radius (or diameter).                              | 26a. See activity # 20.  |

## ACTIVITIES AND MEDIA

1. Addison-Wesley Mathematic Program  
Book 3 (1964) End of chapters (pp. 46-47; 76-77; 112-113; 150-151; 178-179; 200-201; 236-237)  
Book 3 (1968) pp. 44-51; 128-135; 190-197; 246-253.  
  
Teachers Edition (1968) Activities and Follow-Ups, pp. 130-131; 134-135; 250-251.  
Evaluation:  
Observation check since intuitive insight into some geometric ideas was the objective. (100% accuracy).  
  
Seeing Through Arithmetic  
Book 3 Scott Foresman (1963), pp. 524-527.
2. Addison-Wesley Mathematics Program  
Book 4 (1964) End of chapter (pp. 46-47; 90-91; 130-131; 146-147; 160-161; 184-185; 218-219; 238-239; 280-281).  
Activities and follow up check at two pages.  
  
Evaluation-----Same as Book 3  
  
Scott Foresman-Seeing Through Arithmetic  
(1968) pp. 508-509.
3. Addison-Wesley Mathematics Program  
Book 5 (1964) End of chapter (pp. 146-167)  
Book 5 (1968) ( pp. 76-101-also includes measurement).  
Book 5 (1965) Workbook (pp. 44-45).  
Book 5 (1968) Workbook (pp. 91-98- also includes measurement).  
  
Evaluation:  
Book 5 (1964) Chapter review p. 167.  
Book 5 (1968) Day to day observation and specific testing of their application to practical problems.  
  
Seeing Through Arithmetic  
Scott Foresman--See index Book 5.
4. Addison-Wesley Mathematics Program  
Book 6 (1964) pp. 188-215.  
Book 6 (1964) pp. 95-123 Also, includes measurement.  
See same pages for activities and follow-ups.  
Book 6 (1965) Workbook pp. 59-67.  
Book 6 (1968) Workbook pp. 89-96 (includes measurement)  
  
Evaluation:  
Special care to evaluate the children on a day-to-day basis as they study this section. (100% accuracy).  
  
Seeing Through Arithmetic  
Book 6 (1963) See index.  
  
L.W. Singer Co., Sets and Numbers,  
Book 6 (1966) pp. 392-409 shows how to construct the figures of plane geometry.



## MEDIA

5. Transparencies:  
3M no. 26-S pythagorean Theorem.  
3M no. 19 Introducing Geometric Figures
6. Charts:
7. Geo-boards and Geo-blocks
8. Things to do with a Geo-board Activities.
9. Tangram Packet
10. Filmstrips:  
Introduction to Plane Geometry (no. 60)
11. Cyclo. Teacher, Mathematics Cycles.  
Geometry:  
M-60 Recognizing Shapes  
M-61 Perimeter of Rectangles and Squares.  
M-62 Areas of Rectangles.  
M-63 Perimeter of Triangles;  
M-64 Areas of Triangles.  
M-65 Circumferences.  
M-66 Areas of Circles.

## TEACHER'S READING REFERENCES

1. Walter, Marion: "A Second Example of Informal Geometry: Milk Cartons," Arithmetic Teacher, Vol. 16, # 5, May 1969. Describes some work that children can do with milk cartons.
2. Krause, Eugene F.: "Elementary School Metric Geometry," Arithmetic Teacher, Vol. 15, # 8, December 1968. This article explains the concept of measure and illustrates some of its application.
3. Newfeld, K. Allen: "Discovery in Number Operations Through Geometric Constructions," Arithmetic Teacher, Vol 15, # 8, December 1968. A more interesting way to set a group of curious students to exploring the number system and discovering ideas that seem so obvious to them.
4. Complo, Sister Jannita Marie, I.H.M.; "Teaching Geometry Through Creative Movement," Arithmetic Teacher, Vol. 14, # 2, November 1967. Creative movement is the child's creative interpretation of thoughts and feelings through the use of his body. Recommended for primary grades.
5. Smith, Lewis B. "Pegboard Geometry", Arithmetic Teacher, Vol. 12, #4, April 1965. The pegboard is a successful medium for portraying geometric ideas and for promoting exploration.

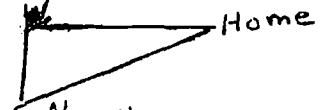
## MEDIA

6. Glenn, William H. and Donovan A. Johnson: The Pythagorean Theorem, Webster Publishing Co., 1960. This booklet has been written on the Pythagorean Theorem so that you may share the pleasure others have had in exploring mathematics.
7. Nuffield Mathematics Project, John Wiley and Sons Inc., New York, 1968, Shape and size. The aim of this project is to devise a contemporary approach for children from 5 to 13. The stress is on "how to learn," not on what to teach.

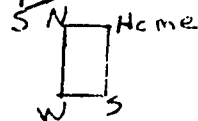
## ACTIVITIES--GEOMETRY

1. Although the history of geometry or "earth measure" closely parallels that of man, efforts to teach this subject effectively in the elementary schools have not been very successful. Although elementary textbooks were filled with geometry in the early part of this century, it almost disappeared during the 1940's and 50's. Now, with the new math, it is returning. The emphasis has been one of "intuitive geometry", or seeing and interpreting forms and shapes around us. The need for extensive proof is delayed until a later time and the students are asked to simply try to know and to appreciate the many varied shapes that they see and feel.

Examples: George went east to the store, then south to a friend's house, then home.



John took three steps to the south, two steps to the west, three steps to the north, and then back home.



Ann spent a quarter at the fair for the merry-go-round. But she didn't go very far--it just went round and round.



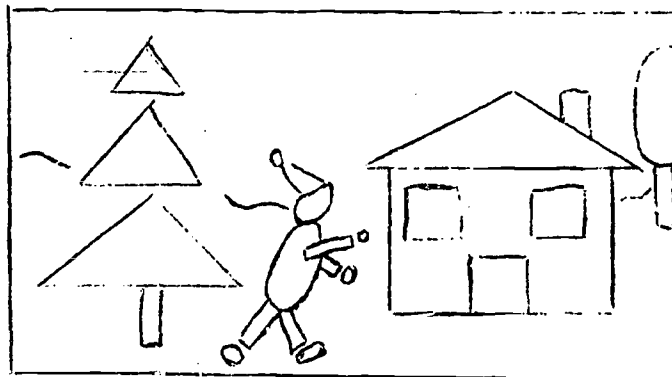
Three steps west, three steps south, three steps east, three steps north and we are back home. We walked a \_\_\_\_\_ today.



### 2. Geometric Art

**Materials:** Have the children help you prepare a box of various geometric shapes out of colored construction paper. Each child will need a generous supply of shapes, paste, crayons, and a sheet of manila paper.

**Directions:** Try to arrange a few of the shapes to resemble something (a fish, a bird, a tree, etc.) When an arrangement is found paste it on the drawing paper. Crayons may be used to add some finishing touches.



3. Geometry is Fun

Have small pieces of wood cut into various geometric shapes, such as a triangle, a square, a rectangle; and solids, such as a sphere, a cone, and a cylinder. Learn the names of these shapes and solids. Also, help the children identify other geometric shapes found around the school building, the school grounds, etc.

4. Geometry Relay Game

Choose teams and have the players sit in rows facing the front of the room. Each player has small cards with the names of the geometric shapes on the cards (square, circle, oval, rectangle, triangle, pentagon, etc.). Place a number of geometric figures made of wood, plastic, or heavy cardboard in a box or a bag and give it to the pupils in the back seats. These pupils choose any one of their figures and hand it to the pupil in the front of them. Each player must select the proper name for the shape. He then passes the figure to the player ahead and reaches for the next figure. The first row finished with all correct answers wins.

To vary the game, each player may be required to identify the shape of the figure before he looks at it.

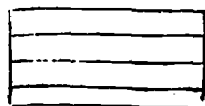
5. Congruence or Similarity Matching

Have each pupil bring a cotton bag, perhaps 6 or 8 inches square. From heavy cardboard, cut small geometric models shaped like triangles, squares, circles, semicircles, etc. Have each pupil put a number of these into his bag. Have a leader select a shape from his bag. Each pupil then draws a model congruent to it. He selects only by his sense of touch. At first, children should have only a few models in the bag. The difficulty increases as the number of models increase.

6. Equal Divisions

The purpose of this exercise is to get pupils to find various ways of dividing a geometric shape into equal parts. Other shapes and other numbers of equal parts may be used to make divisions.

Example:



7. Tell All You Know

Divide the class into teams. Team A selects a geometric shape, such as an equilateral triangle. Team B tries to guess what the shape is by clues given by Team A.

1. It's a simple closed curve.
2. It's a polygon.
3. Its sides are equal.
4. Its angles are equal.

The clues continue until a member of Team B guesses the entire name of the shape. Then Team B selects a shape and gives the clues one at a time, etc. Scoring is optional but the sum of the clues given could be used.

8. Pick Your Trophy

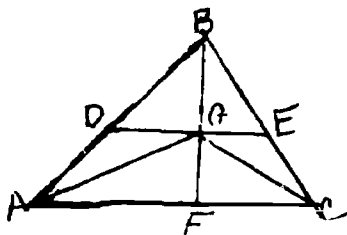
With a piece of masking tape, stick a number of geometric figures on the blackboard. Under each figure write one or more questions about the figure. For example, under the parallelogram.

1. Are any of the angles equal?
2. Are any of the line segments of equal length?
3. Are there right angles in the figure?

Divide the class into two teams. Each player or group of players selects a figure and takes it from the board. To keep the figure (trophy), the pupils must answer all the questions correctly and identify it by name. The team with the greater number of trophies win.

9. How Many Figures Do You See?

Ask the children to count the triangles in this figure. Careful counting should produce 13. To keep an accurate record, use letters like  $\triangle ABC$ ,  $\triangle BGF$ .



10. Blind Geometry

Have the class tie a cloth or handkerchief over their eyes. Separate the boys and girls. Hand the first boy a piece of cardboard shaped like a geometric figure. He then makes some statement about the shape like "It's a simple closed curve." He then passes it to the next boy who gives a clue. Any girl may volunteer a guess by raising her hand, and one guess is allowed for each statement. The number of statements needed before a successful guess is made provides a score for the boys. Any incorrect statement can be overruled by the teacher. All statements are based upon touch. All answers must be based on these statements.

11. Coverall Contest

Materials: Set a flannel board in the front of the room. Cut two corresponding sets of geometric figures from red and blue construction paper. Make a set of cards with the names of the figures on them.

Directions: Display the figures on a table or bulletin board. Divide the class into girls and boys with the red figures for the girls and blue for the boys. In alternate turns, the boys and girls draw a card from the deck, select the shape named from their set of shapes and put it on the flannel board. A certain side of the flannel board should be designated for a team. If they can place the figure without touching another one already in place, they win a point for their team. If the figure touches another one it must be returned to the original supply. The object of the game is to put as many figures as possible on the board.

12. To Tell The Truth

Three pupils sit behind a table. Before each is a card with a name, such as Mr. Triangle. Each pupil announces himself. The class, or a committee from the class, ask a series of questions (5 or 6). Two of the pupils give at least one wrong answer to a question, while the other pupil must give all the correct answers. For example:

Question: Is a triangle a simple closed curve?

Yes, all boundaries are curves. (right)

No, all of its boundaries are line segments. (wrong)

After the players have listened to the three pretenders, they vote on who the "real Mr. Triangle" might be.

13. Geometric Baseball

Materials: Make a set of cards with problems involving geometric situations placed on them. For example: Find the area of an eight-foot square

Directions: Divide the class into teams. One player or the teacher is the umpire. The pitcher (pupils take turns as pitcher) must retire three batters to end an inning. Shuffle the cards and lay them face down on the table. The pitcher draws a card and lets the umpire and the defensive team read the problem before the pitcher makes a pitch. If the pitcher gives a strong clue the umpire calls, "strike". If the clue is weak the umpire calls, "Ball." For example: For the area of an eight-foot square--

It is a measure of a plane region.--strike one

The plane region is a polygon.--ball one

The polygon is a quadrilateral.--strike two

The corner angles are equal.--ball two

The line segments in the polygon are equal.--ball three

The count is now three balls and two strikes. If the pitcher now calls "8 feet", and the batter does not say "64 square feet," the batter strikes out. If he does say, "64 square feet", he gets on first base with a hit. If the pitcher says only "8," the umpire must call "Ball four" unless the batter tries to guess. If he guesses 64 square feet, he gets a hit. If he guesses 64 square inches, a fielder, who also knows the problem, can catch the ball and the player is out. If 64 square inches is not challenged, the batter reaches base on a fielding error. If the batter does not swing, he walks.

## Baseball (cont.)

If a batter "swings" after any clue, he advances to base if he is correct. If he is wrong and is not challenged, he gets to base on an error. Any fielders on the defensive team may challenge the batter. Extra bases may be awarded by the umpire when a fielder makes an error. A "home run" can be declared when the batter gets the answer on the first pitch; its a double if he gets it on the second pitch; anything after that is a single.

### 14. Shapely Art

Start with some geometric shape, such as a circle. Using only that shape, draw some object or idea. If desired, add features, colors, or other trimmings to drawings.

### 15. Vocabulary Game

Prepare a deck of cards, each showing the name of some geometric term or expression commonly used. If this is used as a team game, the cards are shuffled and a student draws a card from the deck and draw or demonstrate that he knows the meaning of the word. A teacher, or a committee of pupils will serve as judge.

Diagonal	=	Diagonal	Area	=	Plane region or <del>area</del>
$\pi$	=	Diameter times $\pi$ = Circumference	Curve	=	A path though a set of points

16. Formulas and tables play an important part in arithmetic. Make a class booklet of some of the ones needed. As a new one is met, it can be added. On 4" x 6" oak-tag sheets, write with a felt pen one formula on each page. Fasten together with large rings and leave where anyone can refer to it easily.

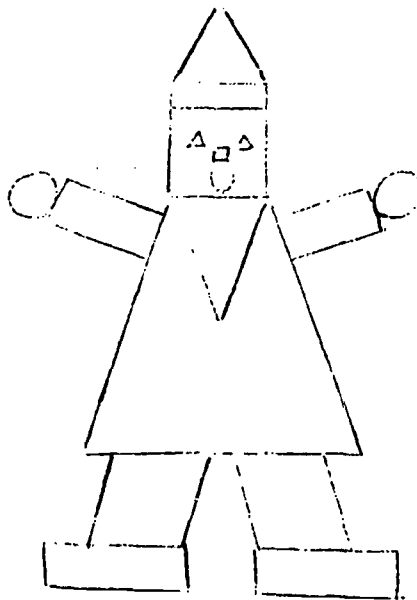
### 17. Geometric Gymnastics

Materials: Make two sets of cards. On each card of one set draw a geometric figure, and on each card of the second set write the name that describes the figure. The second set could also contain three blank cards.

Directions: Divide the name cards evenly between two players. The figure cards are placed face down between the players. Each player takes turns turning up one of the figure cards. If he has a name card that matches it, and if he does so correctly, he claims the figure card; the other player gets a turn. If he cannot match the card, the figure card is returned to the bottom of the pile, face down, and the next player plays. The player with the greater number of figure cards wins.

18. Geo-Santa

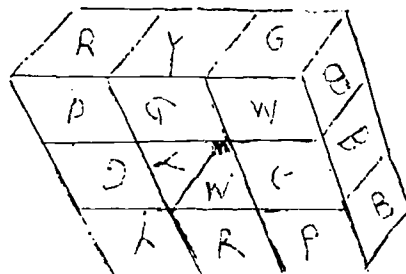
1. Draw the following geometric figures (younger children can draw these shapes easier on graph paper); cut them out; put them together to make a Santa.
2. For Santa's hat, make an EQUILATERAL TRIANGLE measuring 1" on each side. The brim is made from a RECTANGLE that measures  $1\frac{1}{4}$  by 1".
3. Santa's face is made from a SQUARE 1" on each side. Give him two little TRIANGLES for eyes, a little SQUARE for a nose, and a little CIRCLE for a mouth.
4. Make Santa's beard with an ISOSCELES TRIANGLE, measuring 1" on one side,  $1\frac{1}{2}$  inches on each of the other two sides.
5. His body is made from an ISOSCELES TRAPEZOID measuring 3" on the two equal but nonparallel sides, 3" on one parallel base, and 1" on the other base.
6. Santa's arms are outstretched with two RECTANGLES that are 1" long and  $\frac{1}{2}$ " wide.
7. His hands are made from two CIRCLES  $\frac{1}{2}$ " in diameter.
8. Each of his legs is made from a RHOMBUS, 1" on a side.
9. His feet are made from two RECTANGLES,  $1\frac{1}{2}$ " by  $\frac{1}{2}$ ".
10. Give him three CIRCLE buttons and the task is finished.





19. Geometry Problems

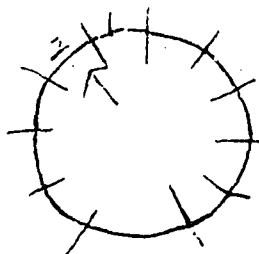
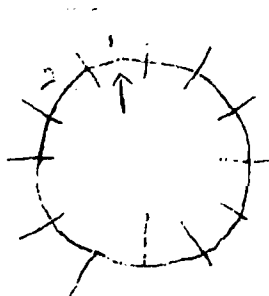
1. An explorer walks one mile south, turns and walks one mile east, turns again and walks one mile due north. He finds himself back where he started. At what location on the earth's surface is this possible? (Answer: at the North Pole or about  $1 \frac{1}{6}$  miles from the South Pole)
2. The blocks of this drawing are identically lettered. Roll them around with your imagination or sketch a pattern so that you can tell the letter that is on the side opposite Y. Opposite G. Opposite W. (Answer: D is opposite Y, R is opposite G, and P is opposite W.)



20. A Disappearing Line

Show the class a circle similar to the one in Figure A below. The circle is cut out so that it is possible to change its position. When the circle is in the position with the arrow pointing to 1, twelve lines can be counted. When the circle is moved so that the arrow points to the numeral 2, as shown in figure B, only eleven lines are visible. One line has disappeared.

The disappearing line problem can be explained by a very basic geometric assumption. When the circle is moved from 1 to 2, not only does one line disappear, but each one is a little longer than the original segments. The total length of the line segments in the first position is equal to the total length of the segments in second position. The geometric principle applied is, "The whole is always equal to the sum of its parts, no matter how the parts are rearranged."



21. A Pentagon-----Cut a rectangular-shaped strip of paper about  $1\frac{1}{2}$  inches wide and 15 inches long. Mark both sides of one end A and both sides of the other end B. Form a loop with the strip by bringing end A over across end B. Then bring end A under end B and through the loop. Pull ends A and B outward until your model forms a pentagon. Press the model tightly to crease the edges. Then trim off the A and B ends of the strip so your model represents a pentagon.

A Hexagon-----Cut two rectangular-shaped strips of paper about  $1\frac{1}{2}$  inches wide and 2 feet long. Mark the ends of one strip A and B. Mark the ends of the other strip C and D. Loop end C under and then over across strip AB. Then loop end A over on top of ends C and D. Bring end D over and across ends A and C and through the loop made by strip AB. Bring end A over across end B. Pull ends A and B together to the right and ends C and D together to the left and crease the edges. Trim off ends A, B, C, and D to obtain your model of a hexagon.

A parallelogram-----Use four cardboard strips to show a parallelogram. Use cardboard strips with the opposite sides of the same length. If you move the sides, you will demonstrate many other parallelograms. The corners may be fastened with paper fasteners.

## 22. Toytown

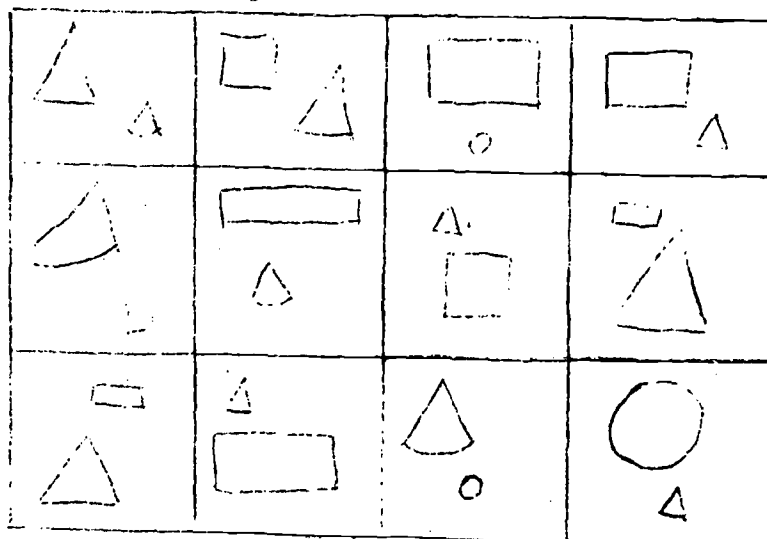
Collect pictures of many different toys from colorful catalogs and mount these on construction paper. Make a bulletin board display of them near the time this project is being used. A list of the shapes for some toys follows.

Toys  
drums  
blocks  
archery set

Shapes They Contain  
circles, cylinders  
squares, rectangles, cubes  
concentric circles

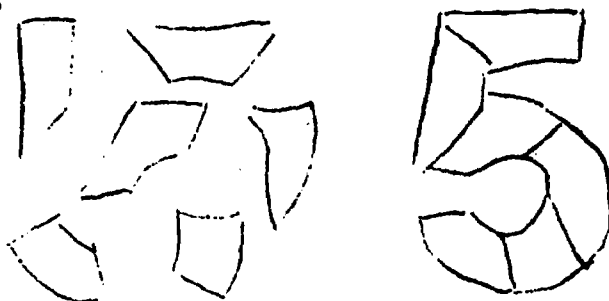
## 23. Geometry shapes Twister

One student spins a spinner, two other students stand, one on either end of the board. The student who is spinning gives the direction. Example: "Put your right hand in a square containing a large blue triangle and a small red cone." The first student to place his hand or foot in the wrong place or to fail loses. For upper grades it would be possible to incorporate more shapes and perhaps put in diagonals to indicate grooves and dots to show texture.



24. Cut-Ups of Numerals and Letters

Let pupils cut apart numerals, letters or geometric shapes into geometric shapes. As they reassemble them in order to get the correct item, they get good practice in discriminating between various shapes and sizes. Some pupils may enjoy trying to construct geometric designs out of the different cut-ups.

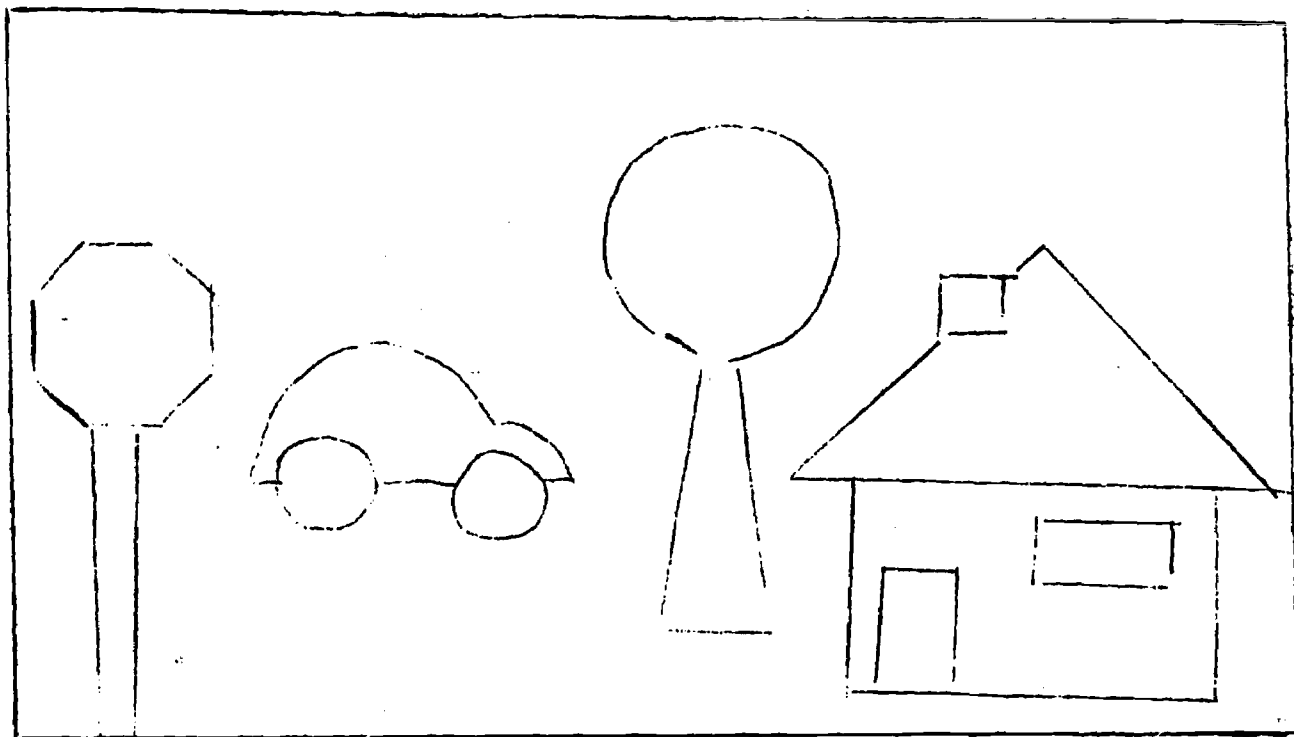


25. Stage a Shape Relay

Scatter a variety of shapes on a table at one side of the room. Lines angles, open and closed curves and rays could also be included. Choose two teams. Name a shape or item. The first child from each team must go to the table and find a sample of the shape which has been named. If he is correct, he goes to the end of his line. Name a different shape to be found by the second pair of contestants, still a different for the third, and so forth.

26. Polygon City

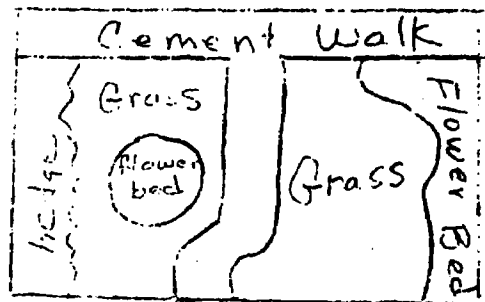
Have the children divide into groups and see which group can make a mural of a city using the most geometric shapes, lines, and angles. They could work with pencils, crayon or construction paper cut-outs. To add variety solid or three-dimensional "sculptures" may be produced by using a material such as styrofoam cut into desired thicknesses.



27. Identifying Shapes, Points and Curves

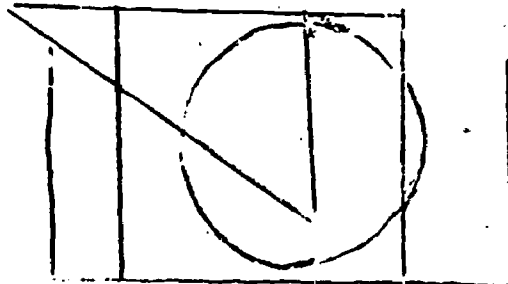
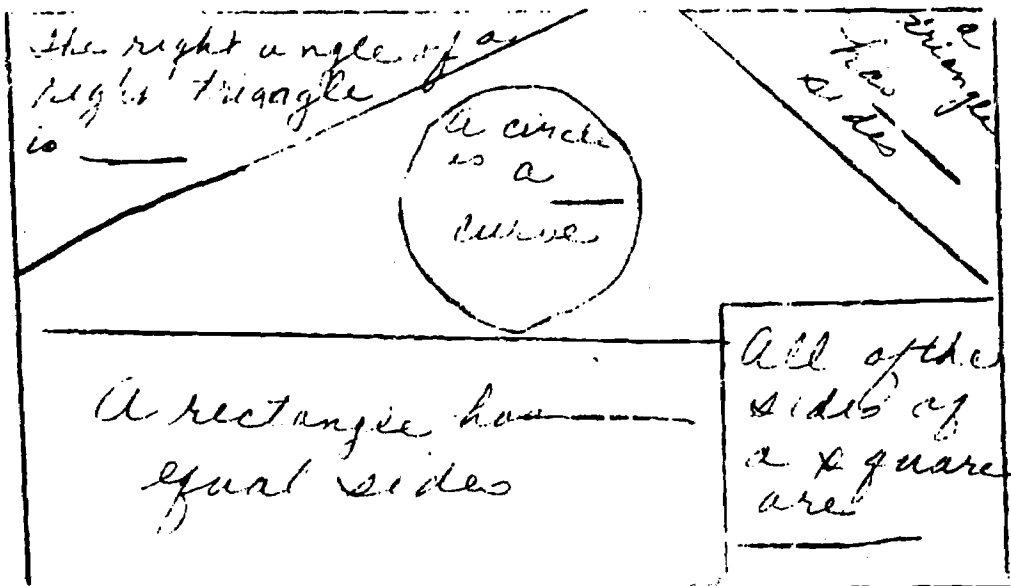
Make two of the following charts and have the children divide into two teams. Set a time limit and see which team can locate the most shapes, points and curves.

A simple example:



28. A Puzzle

Two identical puzzles, made of geometric shapes, are placed on a table. The teams sit around the table and upon their turn the player designates the question (which is written on the puzzle where the shape is to fit) he is going to answer, gives the answer and then, if correct places the appropriate puzzle piece over the question. The team which completes its puzzle first wins.

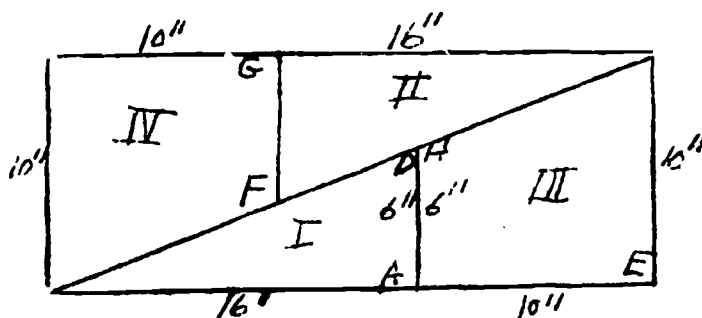
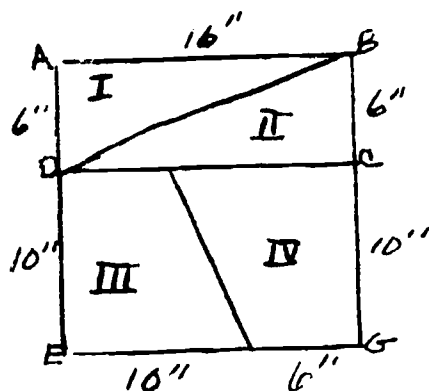


29. An Elusive Area

Draw a square sixteen inches by sixteen inches, and then draw lines to divide it into four parts, as shown below. Tell the class that you will make a drawing that will rearrange the four parts of the square into a rectangle. The second drawing should be like the second one below. Note that the dimensions of the four parts that make up the rectangle match perfectly.

Ask the class to compute the area of the square:  $16 \times 16 = 256$  sq. in. Ask them to compute the area of the rectangle:  $10 \times 26 = 260$  sq. in. Four square inches of area has been created. If you change the rectangle back to a square, the area disappears.

Proof: It was assumed that the lines BD and FH of the square will form a straight line when put together in the rectangle. They do not form a straight line. As a result, the parts do not fit together perfectly and a rectangle cannot be formed.



READING, INTERPRETING AND MAKING  
VARIOUS TYPES OF GRAPHS

VIII. GRAPHS  
Pictorial graphs

K-3-I, G4-S, G5-6-R

- |  |   |
|--|---|
| <p>1. Given a picture graph, the learner can write or state orally three factual statements derived from the study of the graph.</p>   | <p>1a. Show class examples of pictographs collected from national magazines. Pupils will enjoy inventing other simplified pictures appropriate for the pictographs you display. (Suggestion: Pads of ordinary graph paper should be made available to the pupils. This produces neater and clearer work.)</p>   |
| <p>2. The student can write or state orally three factual statements derived from the study of a bar graph. (Horizontal, vertical, and double bar graphs.)</p>                       | <p>2a. Find in newspapers or magazines and bring to class some bar graphs. Be ready to explain to the class what the graphs show.</p> <p>b. See activity #1</p>   |
| <p>3. Given the raw data for a graph of a minimum of five bars, such data in numerical multiples of 5 or 10, the learner can construct a bar graph to illustrate the given data.</p> | <p>3a. Use information from the social studies program or from science that may be organized and shown on a bar graph. Pupils may work individually or in small groups on constructing a graph.</p> <p>b. Pupils may record results for a series of arithmetic tests by making a vertical bar graph. (Then they should compute the mean average.)</p> <p>c. See activity #2</p> |

READING, INTERPRETING, AND MAKING  
VARIOUS TYPES OF GRAPHS

### VIII. GRAPHS

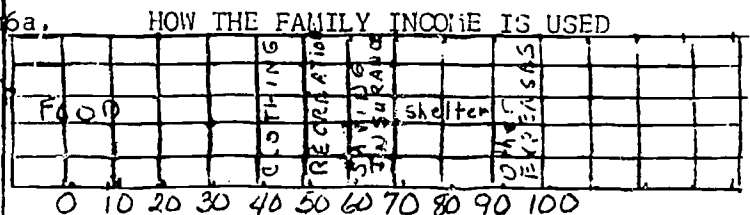
Pictorial graphs

K-3-I, G4-S, G5-6-R

- |  |  |
|--|--|
| 4. The student can write or give orally three factual statements from a study of a given line graph. | 4a. Using the temperature readings taken at five different times during the day, the student can plot these on a line graph. |
|--|--|

- |  |                     |
|--|---------------------|
| 5. Given the raw data for a line graph, the student can construct a line graph to illustrate the given data. | 5a. See activity #5 |
|--|---------------------|

6. Given percentages of total (such as, as percentage of income spent in various ways) and graph paper, the learner can construct a divided bar graph to illustrate the data given.



Read the graph and tell which of the per cents, 24%, 10%, or 5% of \$100, best represents the percent spent for recreation; for shelter.

What decimal tells the per cent of \$100 that was used for food? for clothing? for "other expenses"?

READING, INTERPRETING, AND MAKING  
VARIOUS TYPES OF GRAPHS

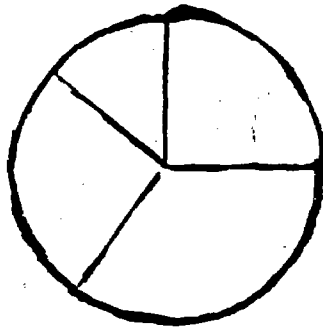
VIII. GRAPHS

Pictorial graphs

K-3-I, G4-S, G5-6-R

7. Given a circle graph and five questions based on the given graph, the student can answer the questions by referring to the graph.

- 7a. Have pupils draw circles on plain paper by means of saucers or other circular objects. Use circles of at least 6" in diameter. Have pupils fold the circle shapes once, crease, and unfold them to obtain a circle shape separated into two equal parts. Use added folds to form four or eight sectors. Coloring various parts in different colors provides a variety of graphs.
- b. Have pupils find circle graphs in newspapers and magazines, bring them to class for discussion.
- c. Answer the questions about the graph showing how Jane spent her 40 day vacation.



- (1) How did she spend the largest part of her vacation?
- (2) About what part of her vacation did she spend at home?
- (3) About how many days of her vacation were spent visiting at camp? at home?
- (4) The time spent at camp was about what fractional part of the time spent at home? of the time spent traveling?



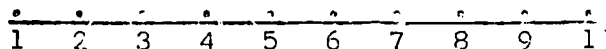
# GRAPHING SETS OF POINTS AND PAIRS OF NUMBERS

## VIII. GRAPHS Coordinate

G5-S, G6-R

8. Given a set of numbers, the learner can mark a point for each number in a set.

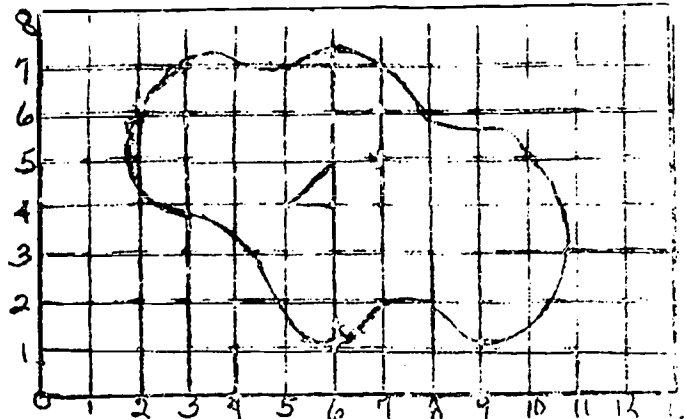
- 8a. This can be done on paper or on the chalkboard.



9. Given a line graph which illustrates a progression of ordered pairs of numerals, the learner can write the set of ordered pairs illustrated by the graph.

- 9a. Let children play the game Find the Treasure. Two children or teams each draw a picture of an island on a coordinate grid. They may "hide" a treasure on the island. The treasure should be identified by three points which may be in a line (vertical, horizontal, or diagonal) or be the corner points of a triangle. Children may take turns calling out names for coordinate points (5,2), and so on, and on another coordinate grid making an X at the point called. The object is to locate the treasure of the other team. Of course, locating the island is helpful. When a called point falls on part of the island, the opponent must say, "You are on my island." When any portion of the treasure is "hit" by a called point, the game is over.

EXAMPLE OF GRID FOR FIND THE TREASURE



# GRAPHING SETS OF POINTS AND PAIRS OF NUMBERS

## VIII. GRAPHS Coordinate

G5-3, G6-R

10. When given a set of ordered pairs and a sheet of graph paper, the learner can construct a line graph to illustrate the set of ordered pairs of numerals.

10a. See activities #3, 4

11. The learner can graph the number pairs for the function from the following table.

11a. Addison-Wesley Book 6  
1964, P. 305 (Graphing functions)

function rule	
$n + 1$	
n	f(n)
0	1
1	2
2	3
3	4
4	5
5	6

# GRAPHING FUNCTIONS

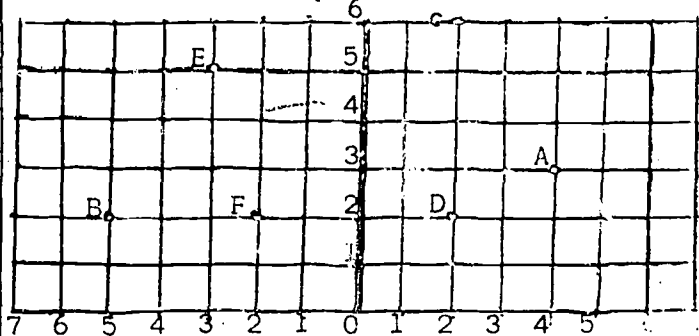
VIII. GRAPHS  
Coordinate

G6-S, G7-R

12. The student can graph the following pairs of integers given in the table below. (Involves negative numbers).

function rule	
n	f(n)
-4	-1
-3	0
-2	1
-1	2
0	3
1	4

- 12a. Use graph paper for the following activity. Pupils are introduced to negative numbers and then are able to label points below 0 on a vertical number-line model. The number for the point located is called its coordinate. Now pupils can use this knowledge in locating points on a grid. (Negative numbers are shown now at the left of 0 on a horizontal model.) Each point on the grid is determined by two numbers or coordinates. The symbols already located on the grid are described below under headings "Over" and "Up".



Write or tell the letter located at these points:

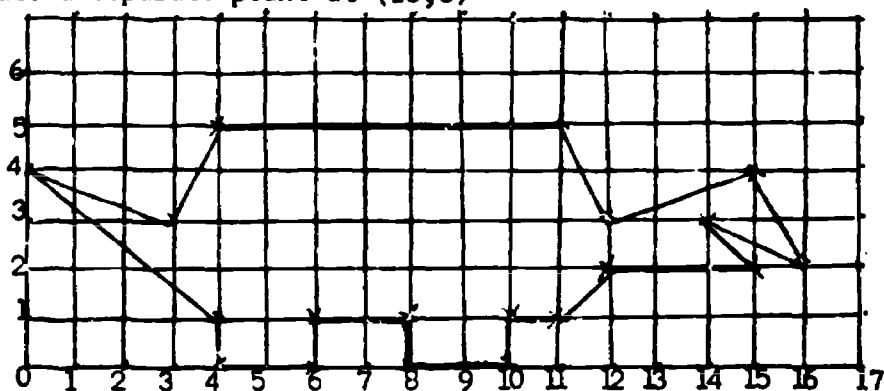
Over	Up
2	6
4	3
-3	5
-5	2
2	2
-2	2

## GRAPH ACTIVITIES

1. The class might construct a chart on the floor by using masking tape for lines. Children might then "walk off" specific points on the graph. This activity, as well as the overall study of graphing, might easily be correlated with map study with emphasis on the lines of latitude and longitude. Before constructing graphs, children should have opportunity to study many meaningful graphs, such as city maps, road maps, and graphs charting rainfall, temperatures, etc.
2. Let each child make a booklet of different kinds of graphs in colors to show his progress in each subject during the month. This teaches the purpose and use of graphs, encourages the pupils to greater effort. They learn about graphs without thinking of them as a dull arithmetic lesson.
3. Locate each of these points and connect the dots in the order given. Use a grid and see what you have drawn.
 

a. (0,4)	f. (15,4)	k. (11,1)	p. (6,1)
b. (3,3)	g. (16,2)	l. (10,1)	q. (6,0)
c. (4,5)	h. (14,3)	m. (10,0)	r. (4,0)
d. (11,5)	i. (15,2)	n. (8,0)	s. (4,1)
e. (12,3)	j. (12,2)	o. (8,1)	t. (0,4)

Locate a separate point at (15,3)



4. A student can draw a picture of his own and give a list of ordered pairs of numbers which a classmate may follow to put the picture on a grid.
5. Other suggested activities:  
Baseball fans will find a wealth of data in baseball handbooks and the "ups and downs" of the school's own teams are sources of statistical materials.

Find the height of each of five pupils in your class. Make a table to show the information and then make a vertical bar graph to go with the table. Label the vertical scale so that each unit represents 10 (inches)

Using an almanac, make a line graph to show the population of the United States for each of four census years.

Make a graph showing the speed of travel of various bullets, missiles, or planes. Check data on these speeds in an encyclopedia.

## MEDIA

1. Addison-Wesley Mathematics Program  
Book 5 (1964) P. 302-311  
Book 5 (1968) P. 304-313  
  
Workbook (1965) P. 94-96  
Workbook (1968) P. 105-108  
  
Duplicator Masters (1969) Grids P. 54-56 Grade 5
2. Book 6 (1964) P. 298-307  
Book 6 (1968) P. 302-311  
  
Workbook (1965) P. 94-96  
Workbook (1968) P. 106-109
3. Scott Foresman-Seeing Through Arithmetic  
Book 5 (1963) P. 510-517  
Book 6 (1963) P. 266-269, 270-272
4. Transparencies:
5. Charts:
6. Filmstrips:
7. S.R.A. Graphs and Picture Study Skills Kit (Coffeen)

## TEACHERS' READING REFERENCES:

1. Heard, Ida Mae: "Making and Using Graphs in the Kindergarten Mathematics Program", The Arithmetic Teacher, Vol. 15, No. 6, October 1968. Describe and illustrates ways in which graphs might be used with young children.
2. Pierson, Robert C.: "Elementary Graphing Experiences", The Arithmetic Teacher, Vol. 16, No. 3, March 1969. Exercises in graphing to help guide the teacher.
3. Schell, Leo M.: "Horizontal Enrichment with Graphs" The Arithmetic Teacher, Vol. 16, No. 3, December 1967. Lessons designed to supplement a unit on graphs, where the plotted points do not lie on a straight or a smooth line

PREFACE  
to  
JUNIOR HIGH SECTION

The Junior High attempts to be the coordinating link between the elementary math student using Addison and Wesley texts and the high school math student using Houghton Mifflin texts. For the 7th and 8th grade High and Average groups texts used are Holt, Rinehart Winston's Elementary Mathematics, Patterns and Structures, 7 and 8. The numerous "Brainteasers" are used for extension and enrichment in both grades. Cumulative Tests in the front of both texts could supply good evaluation. The abbreviation HRW 7 or HRW 8 indicates these texts. When reference is made to other texts, these listings indicate publisher first, author second, and title third (in caps).

The Addison Wesley set of Basic Modern Mathematics, Courses 1 and 2, and Modern General Mathematics (Course 3), listed as Books 1651, 1661, and 1671, respectively, are used for Remedial 7 and 8, where the main objective is to improve computational skills and to increase understanding of fundamental number operations. Particular emphasis is directed to where the various kinds of mathematics are used in daily life, and units are set up, outside the book, utilizing these relevancies to daily living and often culminating in field trips, etc. Reading is de-emphasized and manipulation and actual performance skills are encouraged. Wide variety and short lessons are effective for this particular group of learners. AW 1651, AW 1661, and AW 1671 are the abbreviations for these three overlapping texts used in Remedial 7th and 8th grade math.

We would like to especially recommend two books as exceptionally useful. Diagnostic tests and interpretations, plus what corrective phase may be possible, for addition, subtraction, multiplication, division, decimals, number sentences, ratio and proportion, percent, word problems, measurement, and practical applications are available in:

Holt, Rinehart and Winston, Bernstein and Wells:  
TROUBLESHOOTING MATHEMATICS SKILLS, Teacher's  
Edition, copyright 1969, 1963.

Enrichment activities for the students who progress more rapidly than their classmates may be found in

National Council of Teachers of Mathematics,  
ENRICHMENT MATHEMATICS FOR THE GRADES,  
copyright 1963, pages 207-311.

These activities include puzzles, unit fractions, divisibility rule, numeration systems, numbers and games, sets, distances, mathematical systems, logic, and geometry.

Further Supplementary Activities are listed in the pink sheets following each section.

REVIEW SET UNION & INTERSECTION;  
INTRODUCE COMPLEMENTS

I Sets

7 S

1. Given two sets, student can construct union of the two sets.
2. Given two sets, student can construct intersection of the two sets.
3. Given a set and a subset, the student can identify in writing the complement of the subset.

HRW 7, pp. 113-123; 291-292

Readiness: Recognition of kinds of sets and set terminology, including diagnostic matching.

CONCEPT OF SOLUTION SETS AND THEIR USE IN  
PROBLEM-SOLVING

I. Sets

7 S

1. Using proper set notation, the student will correctly identify, in writing, solution sets for 8 out of 10 given closed and open sentences, some of which may be word problems.

Readiness: Recognition of kinds of sets and set terminology, including diagnostic matching.

HRW 7, pp. 43-45

Universal or replacement set limits solution.

Learns to solve closed sentences.  
Learns to solve open sentences.

Learns to recognize cases where solution sets can be empty sets or infinite sets.

Must be SURE to use SET NOTATION in answer.

CONCEPT OF SOLUTION SETS AND THEIR USE  
IN PROBLEM-SOLVING

I. Sets

7 S

1. Using proper set notation, the student will correctly identify in writing solution sets for 8 out of 10 given closed and open sentences, some of which may be word problems.

Readiness: Student must be familiar with set concept and set notation, and with open and closed sentences.

HRM, pp. 43-45

Universal or replacement set limits solution.

Learns to solve closed sentences.

Learns to solve open sentences.

Learns to recognize cases where solution sets can be empty sets or infinite sets.

Must be SURE to use SET NOTATION in answer.



CONCEPTS OF DENSITY AND BETWEENNESS

I. Sets

7 S

1. When given a list of 6 sets, the student will correctly identify in writing 5 out of 6 dense sets.
2. When given a particular set, the student is able to compute the number which is a given distance between two members of the set.

HRW 7, pp. 181-193

Betweenness does not imply "half-way-between."

Density can change depending on sets given, from 0 to infinity.

HRW 7, p. 185

Techniques for finding a number which is between two other numbers.

HRW 7, pp. 182-185; 391-393

Arithmetic average or mean.

High groups will attain pp. 391-393 of text, on median, mean, and mode.

STRENGTHEN CONCEPT OF ARITHMETIC MEAN BY  
BY ADDING "MEAN," "MEDIAN," AND "MODE."

I. Sets

7 S; 8 R

1. When given 10 sets of data (such as test scores), the student can compute the arithmetic mean, median, and mode, some of these sets being in word problem form, with 70% accuracy.

Readiness: Concepts of average and betweenness.

HRW 7, pp. 182-185

HRW 8, pp. 40-42; 90-91 on progressions.

Webster, Osborne et al, EXPLORING ARITHMETIC 8, pp. 212-213

Ginn, Buswell et al, TEACHING ARITHMETIC WE NEED 8, pp. 154-155.

HRW, Nichols: PRE-ALGEBRA MATHEMATICS, pp. 126-129.

Harcourt, Brace & World, Clark: GROWTH IN ARITHMETIC 8, pp. 327-337 (Especially good and can lead into statistical graphs; good for everyday usage.)

STUDENT IS ABLE TO GRAPH A SOLUTION SET ON  
THE NUMBER LINE

I. Sets

8 S

1. Given 5 mathematical sentences, the student can solve and graph the solution set on the number line with 80% accuracy (4 out of 5).

Readiness: Student is familiar with problem-solving of mathematical sentences, set operation, and the number line.

HRW 8, pp. 284-290

Student is made aware which notations limit the solution set and how to graph these.

Student learns to solve and graph problems involving more than one condition.

Enrichment - "The Arithmetic Teacher," Vol. 13, No. 2, Feb., 1966, pp. 98-99.

HRW, PRE-ALGEBRA MATHEMATICS, c 1965, pp. 430-446 on Graphing.

FOR FURTHER GRAPHING OF SETS, 8th grade, SEE  
GEOMETRY & ANALYTIC GEOMETRY(Parabola) Sections

STUDENT IS ABLE TO RECOGNIZE THE NEAREST INTEGER  
WHICH IS NOT GREATER THAN THE EXPRESSION IN HEAVY  
OR BOLDFACE BRACKETS

III. Estimation

8 S

1. Given 5 bracketed expressions, the student will correctly identify in writing the nearest integer not greater than the bracketed expression, in 4 cases out of the 5.

Readiness: Student must be familiar with concepts of less-than or equal-to, and with place value.

HRW 8, pp. 292-295

Student is made aware that this technique may be useful in estimation problems.

I. SETS (Functions and  
Operations)

**SUPPLEMENTARY ACTIVITIES**

**FILM LOOP "OPERATIONS"**

Encyclopedia Britannica  
Super 8 MM Color No. S-80183 \$22.00  
or  
8 MM Color No. R-80183 \$20.00  
Concepts of Binary and Unary Operation.

**FILM LOOP - "OPPOSITES"**

Encyclopedia Britannica  
Super 8 MM Color No. S-80178 \$22.00  
or  
8 MM Color No. R-80178 \$20.00  
Real and abstract situations using  
Operation and its inverses.

**FILM LOOP - "M(ANY) SOLUTIONS"**

Encyclopedia Britannica  
Super 8 MM Color S-80166 \$22.00  
or  
8 MM Color R-80166 \$20.00  
Sets, Reasoning  
Problems have differing numbers of  
solutions depending on the data.

**FILM LOOP - "INTRODUCING ORDER & SIZE"**

Encyclopedia Britannica  
Super 8 MM Color No. S-80163 \$22.00  
or  
8 MM Color No. R-80163 \$20.00  
Cartoon situation to illustrate  
commutative property and basic  
inequalities.

**'SETS, SENTENCES AND OPERATIONS'**

by Donovan A. Johnson & M. H. Glenn  
Paperback, c 1960.  
Webster Publishing Co.  
Quite a few Venn diagrams illustrating  
the various operations of sets  
and possible applications.

Laidlaw, MATHEMATICS 8, McSwain et al,  
c. 1965., pp. 61-72.

Chapter 4 - Sentences about Numbers,  
Order Relations, Open Sentences,  
Replacement Sets, Subsets, Disjoint  
Sets, Solution Sets, Empty Sets.  
Chapter 2, p. 24 - Set Notation and  
Matching Sets.

MODERN MATHEMATICS, Teacher's Edition,  
Singer/Random House, c. 1965, 1961  
Paperback.

When additional practice is needed in  
the items listed below, this book is  
handy to have around.

1. Ancient Systems of Numeration
2. Cardinal and Ordinal Numbers
3. Place Value
4. Other Numeration Bases
5. Number Patterns (Squares,  
Reciprocals, etc.)
6. Number Line
7. Principles - Closure, Commutative,  
order of Operation, etc.
8. Sets
9. Ordered Pairs
10. Sentences and variables
11. Graphs of sentences.

**RENTAL FILM - INTERSECTION OF SETS**

15 min. Color \$5.00 for 1-3 days

U. of Mo. 1968 'New Films'

Describes meaning of set and subset and  
illustrates intersection of sets, showing  
various degrees of overlapping between  
sets.

REINFORCE AND REVIEW NUMBER THEORY CONCERNING  
ODD AND EVEN NUMBERS, AND CLOCK ARITHMETIC

II. C5. Place Value

7-8 REMEDIAL

1. Given a list of five digits, the student can discriminate between odd and even numbers with 80% accuracy.
2. Given the modified number systems of 4 and 12, the student will solve clock arithmetic equations with 70% accuracy.

Readiness: Familiarity with basic combinations.

AW 1651, pp. 246-248

AW 1661, pp. 216-217

AW 1671, pp. 176-181

REVIEW AND REINFORCE NUMBER THEORY CONCERNING  
FACTORS, MULTIPLES, AND PRIMES

II. E. Factors

8 Remedial

1. Given a list of ten numbers, the student will factor, find multiples or primes, as indicated, with 60% accuracy.

Readiness: Basic multiplication and division facts.

AW 1651, pp. 249-259

AW 1661, pp. 218-233

AW 1671, pp. 249-259

Factor Trees  
Function Machines  
Sieve of Eratosthanes

Prime Factorization = Complete Factorization

Composite = Non-Prime Number  
Cyclo-Teacher M 79-80; M77.

Demonstrate Least Common Multiple and Greatest Common Factor as intersection of two sets.

STUDENT IS ABLE TO COMPUTE ROMAN NUMERALS  
USING THE MULTIPLICATIVE BAR AND THE ADDITIVE  
AND SUBTRACTIVE PRINCIPLES

## II. Roman Numerals

7 S

1. Student will correctly compute values of 2 out of 3 given Roman numerals utilizing the overhead bar.
2. Student will correctly convert 8 out of 10 given numbers into Roman numerals by proper use of additive and subtractive principles.

HRW 7, pp. 9-11

Introduction of overhead bar, which means multiplying by 1000.

Overhead projector demonstrations on proper uses of subtractive principle (IV and IX, not IL), and of additive principle (VI, VII, VIII but not VIIII), indicating limitations.

Transparencies on Roman numerals.

Contrast unwieldiness of Roman numeral calculations with our own Arabic system.

STUDENT WILL BE ABLE TO RECOGNIZE EXPONENTS  
& RE-NAME NUMBERS IN EXPONENTIAL NOTATION

## II. Exponents

7 S

1. Student will identify in writing exponents in 5 given expressions.
2. Student will be able to re-write a given number (like 694) in powers of a given exponent.

Readiness: Students must be familiar with expanded notation in Base 10.

HRW 7, pp. 15-34.

By transparencies and overhead projector demonstrations, identifications of "exponent," "base factor," and "power" are made.

(We prefer to use "base factor" to eliminate confusion regarding other-base numerations and also to remind student of the function of an exponent.)

Practice in writing numbers in expanded notation, first in Base-10 and then in other specified bases.

SEE ALSO: Addison Wesley 6th grade text, pp. 10-13.

STUDENT WILL BE ABLE TO CONVERT BASE-10 NO.  
INTO A BASE OTHER THAN 10 (AND REVERSE) AND  
WILL BE ABLE TO PERFORM ADDITION AND SUBTRAC-  
TION OPERATIONS USING THESE BASES

II. Other No. Bases

7 S

1. The student will be able to convert 5 given base-10 numbers into a given base, and will be able to convert 5 given other-than-base-10 numbers into base-10, with 80% accuracy (4 out of 5).

Readiness: Student is able to re-name base-10 numbers in expanded notation and in exponential notation.

HRW7, pp. 35-98

Money concepts (pennies, nickels, quarters for Base 5)

Abacus - Transmobile or other.

Overhead transparencies.

STUDENT WILL DEVELOP EXPONENTIAL PATTERN  
LEADING TO NEGATIVE EXPONENTS

II. Exponents

8 S

1. Given a list of 10 numerals, student will successfully compute the value for exponential forms, and develop the exponential form from the given fractional types, with 80% accuracy (8 out of 10). These numerals will include some with negative exponents.

Readiness: Working knowledge of expanded notation of exponents.

HRW 8, pp. 70-74.

Continues pattern from positive exponents.

Practice deriving values for negative exponents and converting given number into negative exponent form.

Addison-Wesley, Wilcox: MATHEMATICS, A MODERN APPROACH, #8677, pp. 223-227.

HRW, PRE-ALGEBRA MATHEMATICS, c 1965, p. 4; 13-16.

Allyn & Bacon, Stein: FUNDAMENTALS OF MATHEMATICS c 1968, p. 30

HRW, Nichols: MODERN ELEMENTARY ALGEBRA, c 1965, 1961; pp. 312-321.

## REVIEW OTHER NUMBER BASES

## II. Other Number Bases

### 7-8 REMEDIAL

1. Given a list of numerals selected from base ten and other number bases, the student will correctly pair 60% (6 out of 10) of the given list.

Readiness: Working knowledge of expanded notation in base 10 and familiarity with names for numerals.

AW 1661, pp. 22-25 (base 4 only)

AW 1671, pp. 18-25 (Base 4,6; Roman,others)

Transparencies for Roman Numerals

Base 4 Bingo

Cyclo-Teacher M-46,47,48 for Roman no.

Make pegboard game for other base no.

Make "Concentration" game for pairing, with base 10 and some other base.

## REVIEW READING AND WRITING LARGE NUMBERS

## II. Recognition

### 7-8 REMEDIAL

1. Given a list of ten numerals written in decimal notation and in English, the student will correctly pair 70% (7 out of 10).

Readiness: Concept of place value up to thousands.

AW 1651, pp. 1-19.

AW 1661, pp. 1-27.

AW 1671, pp. 1-29; p. 64.

Transparencies on abacus.

Students make own abacus (straws-lifesavers)

Transmobile (overhead abacus)

"Rounding numbers" - reinforce concept of "more than" and "less than." ( $>$ ,  $<$ )

Student realizes places can be indicated by exponents.

Present exponents as "easy way" to write astronomical numbers.

STUDENT WILL BE ABLE TO RE-NAME BASE-10(DECIMAL) NUMBERS IN SCIENTIFIC NOTATION AND WILL BE ABLE TO DO MULTIPLICATION AND DIVISION INVOLVING THESE TERMS

## II. Scientific Notation

8 S

1. Given a list of Base 10 (Decimal) numbers, the student will be able to re-name 4 out of 5 in correct scientific notation, using both positive and negative exponents.

2. Given a list of 10 indicated calculations, student will correctly compute 8 out of 10, utilizing scientific notation, and giving the answer in proper scientific notation. Some of the "givens" may be in word problem form.

Readiness: Student will be familiar with expanded notation, with exponential notation, and with computations involving both negative and positive exponents.

HRW 8, pp. 75-100

Student is made aware of the wide usefulness of scientific notation, from micro- to astronomical measurements.

Addison-Wesley 8677, Malcox: MATHEMATICS, A MODERN APPROACH, pp. 228-231.

Allyn & Bacon, Stein: FUNDAMENTALS OF MATHEMATICS c 1968, pp. 135-136.

Houghton-Mifflin, Dolciani et al, MODERN SCHOOL MATHEMATICS, Chapter 6, pp. 153-170. This is an especially good resource for the complex and often difficult computation of scientific notation. We think this one is especially helpful and it is also the predecessor of the Algebra book series used in high school.

HRW, Nichols: MODERN ELEMENTARY ALGEBRA c 1965, 1961, pp. 339-340

ADDITION, SUBTRACTION, AND MULTIPLICATION IN MODULAR (CLOCK) ARITHMETIC IN 3-NUMBER, 4-NUMBER, 5-NUMBER, AND 6-NUMBER SYSTEMS

## II. Clock Arithmetic

8 S

1. Given 10 indicated operations in modular systems (addition, subtraction, or multiplication), student will correctly solve 8 out of the 10.

Readiness: Whole number fundamental operations.

HRW 8, pp. 2-25

Clock transparencies for overhead.

Construct addition and multiplication tables for each system, treating subtraction as inverse of addition.

SEE ALSO: Addison-Wesley Grade V Book, c 1969, pp. 318-319.



THE STUDENT WILL BE ABLE TO DETERMINE ABSOLUTE VALUES OF POSITIVE AND NEGATIVE NUMBERS

II. Absolute Value

8 S

1. Given a list of 5 positive and negative numbers, the student can write the absolute value for each signed number with 80% accuracy (4 out of 5).

Readiness: Student must be familiar with directed rational numbers.

HRM8, pp. 212-213

Student learns symbol for absolute value and that all absolute values are +.

HRW 8, pp. 282-284

Student learns to solve equations involving absolute value.

HRW 8, p. 282

Student learns to recognize that absolute value of 0 is 0.

STUDENT WILL ACQUIRE A WORKING KNOWLEDGE OF PER CENT.

III. Per Cent

7 S

1. Given 10 numerals (some kind of fractions), the student is able to correctly write the other two equivalent names.
2. When given 10 problems of various practical applications of percent (including word problems), the student will correctly solve 7 out of the 10.

Readiness: Good understanding of decimals is necessary, as well as familiarity with common fractions and with proportion.

HRW7, pp. 227-264.

Establish meaning of percent together with ability to re-name percent as decimal and as common fraction.

Student is introduced to problem-solving in percents by double ratio (proportion) method.

Student is made aware of various applications of percent by considerable practice in solving both empirical and word problems.

HRW, PRE-ALGEBRA MATHEMATICS c 1965, pp. 160-183.

Allyn & Bacon, Stein: FUNDAMENTALS OF MATHEMATICS c 1968, pp. 149-171.

HRW, Nichols, MODERN ELEMENTARY ALGEBRA c 1965, 1961, pp. 223-224.

REINFORCE THE BASIC PROPERTIES FOR WHOLE NUMBERS

II. F. Properties

7-8 REMEDIAL

1. Given ten indicated operations, the student will be able to match the associative, commutative, and distributive properties with examples of each, correctly answering 7 of the ten.

AW 1651, pp. 156-175

AW 1661 - No pages

AW 1671 - pp. 53-69

Readiness: Basic addition and multiplication skills.

STUDENT IS INTRODUCED TO CLOSURE PROPERTY AND CAN RECOGNIZE TO WHICH OPERATIONS IT APPLIES.

II. Closure Property

7 S and 8 S

1. Given 5 sets and an accompanying operation for each, the student can identify which sets are closed under that operation, with 80% efficiency.

HRW 7, pp. 158-161

HRW 8, pp. 228, 20, 21, 377

Readiness: Familiarity with set concepts and operations.

STUDENT IS ABLE TO RECOGNIZE A RECIPROCAL AS  
A MULTIPLICATIVE INVERSE AND THAT A NUMBER  
TIMES ITS MULTIPLICATIVE INVERSE = 1.

II. Inverse Property

8 S

1. In a given list of 10 numbers, the student will write the correct multiplicative inverses for 8 out of the 10.

Readiness: Familiarity with reciprocal.

HRW 8, p. 252

Remind students to change decimals into common fractions for quick determination of reciprocal.

Student learns to select multiplicative inverse of directed numbers.

Student learns which numbers are their own reciprocals.

## II. Numeration and Number Theory

### SUPPLEMENTARY ACTIVITIES

#### ODD AND EVEN TIC-TAC-TOE

Instead of X and O, players use odd numbers and even numbers. Since there is one more odd digit, the odd digit player begins first. Symbols are alternated in successive games. Each number can be used only once. Player who completes a row, column, or diagonal totaling 15 is the winner! Game is a draw if neither completes 15.

#### MATHEMATICAL SEARCH

- \* Arith. Teacher, Vol. 16, No. 1, Jan. 1969, p. 64.
- Math. Teacher Vol. LXI, No. 5, May 1968
- Math. Teacher Vol. LX, No. 4, April 1967, by Robert E. Reys, 'Mathematical Word Search.'

\* has at least 75 words, all written forward, vertically, horizontally, or diagonally. One word appears 3 times but counts only once. Both puzzle and list of words are printed. Other such puzzles can be made up by putting in the words you want found and then just filling in remaining spaces.

#### NUMBER NAMES GAME

(Could be used for either whole numbers or for types of fractions.) Play like 'Fish' or 'Rummy.' Make at least four cards having different names for the same number. Deal 6 cards to each player and put the rest in the center. 'Books' of any three names for the same number are laid down for a point score. Person scoring most points within specified time is winner.

#### ROMAN NUMERAL RULES

'The Key to Roman Numerals,' by Keith Breithaupt, Arith. Tchr. April 1968, p. 374.  
Summarizes Roman Numeral rules and principles as few textbooks do.

BINGO-TYPE GAMES by Dr. Jo Phillips, Dec. 1967 Instructor, p. 89.

Easiest version - child required to match exactly the pattern on the caller's card with that on a square on his paper. Winner has his whole card covered. The caller should have in his pile some cards which do not match any pattern on children's cards.

Harder: Children cover a square which has same number of dots as caller's card, whether or not these dots are in the same pattern.

Still harder: Children cover a number of dots which is associated with the numeral on the caller's card.

Innumerable variations: Answers to addition, subtraction, multiplication, or division examples. Scoring can be altered to allow bonus points to player who covers a row of even numbers or all prime numbers, or all numbers which are multiples of 7, per maturity level. Could be also work with fractions, geometry, even to matching properties (has exactly two parallel sides; has at least one line of symmetry, etc.).

#### GREATEST COMMON FACTOR

Finding the greatest common factor with the number line. Arith. Teacher Vol. 12, No. 6, Oct. 1965, p. 455.

#### FILM LOOP - 'WHAT DOES 10 MEAN?'

Encyclopedia Britannica  
Super 8 M. Color No. S-80176 \$22.00  
or  
8 M. Color No. R-80176 \$20.00

#### SCIENTIFIC NOTATION

Laidlaw, MATHEMATICS 8, McSwain et al, c. 1965, pp. 281-289.  
Chapter 15 - Scientific Notation, Computing with Large Numbers, Computing with Small Numbers... includes word problems.

II. Numeration and  
Number Theory  
-- page 2.

SUPPLEMENTARY ACTIVITIES

TRANSMOBILE

Abacus for Overhead  
Transmobile No. TM-2  
Weber Costello, Chicago  
Price:

Use for Addition, other bases,  
place value, subtraction, frac-  
tional parts, possibly multiplica-  
tion. Division is difficult on  
abacus. Use for reading and writ-  
ing of numbers, work with decimals,  
etc.

APRIL FOOL ARITHMETIC  
(Bases Other than 10)  
by Paul R. Neuriter, Apr. 1965  
Instructor, p. 17.

Baa baa black sheep,  
Have you any wool?  
Yes sir, yes sir,  
Zopple zop bags full.

Let zop be zero, zoggle be 1,  
and ziggle be 2 for Ternary  
System. (Any nonsense words  
would do.)

<u>Decimal</u>	<u>Ternary</u>
0	zop
1	zoggle
2	ziggle
10	zoggle zop
11	zoggle zoggle
12	zoggle ziggle
20	ziggle zop
21	ziggle zoggle
22	ziggle ziggle
100	zoggle zop zop
101	zoggle zop zoggle
102	zoggle zop ziggle
110	zoggle zoggle zop
etc. to	
220	ziggle ziggle zop

BINARY SYSTEM OR BASE

Use a set of Christmas tree lights (differ-  
ent colors) to represent the 64, 32, 16, 8,  
4, 2, 0 places.

from Webster Publishing Co.'s pamphlet,  
'Understanding Numeration Systems.'  
(Coffeen has copy)  
Also good for exercises in Base 10 to Base  
2 and vice versa conversions.

NIM - a Binary Number Game.  
p. 41 of Webster Co. pamphlet, 'Understand-  
ing Numeration Systems.'  
Pick up counters - sticks, matches, tooth-  
picks, beads, coins, etc.  
P. 48-49 of Webster pamphlet is good review  
for all bases. See also for supplementary  
exercises.

FILM LOOP - 'SYSTEM OF TWOS'  
Encyclopedia Britannica  
Super 8 MM Color No. S-80085 \$17.60  
or  
8 MM Color No. R-80085 \$16.00  
Uses only two numerals, '1' and '0' to  
count.

MODERN MATHEMATICS, Teacher's Edition, Paper-  
back, Singer/Random House, c 1965, 1961.  
Good for extra practice on  
Ancient Systems of Numeration  
Place Value  
Other Numeration Bases

THE TWIST by Marjorie Hughes, Arith.Tchr.,  
March 1964, pp. 204-5.  
'If someone told you that with a simple  
twist you could banish three-fourths of  
all your difficulties in long division, in  
hard multiplication, in addition of uneven  
decimals, and probably in comprehension of  
borrowing and carrying, wouldn't you at  
least give it a try?'

(continued on next page)

THE TWIST (continued)

MERELY TURN THE SHEET OF THEME  
PAPER 90° so that the holes are  
at the top of the page and the lines  
are running vertically! Place  
value is so important, that students  
may well use lines to help them  
separate places properly. The list  
of 6 advantages given by the author  
make sense!

\* \* \* \*

Bulletin Board Ideas for Bases  
Other Than 10

Page 121 of March 1968 Instructor  
by Sister Mary Caroline SSND, St.  
Vincent de Paul School, Milwaukee,  
Wisconsin.

This article shows a Base 8 number  
arranged across a football, on a  
basketball, and on a soccer ball.  
Naturally it is titled "On the  
Ball."

BASES OTHER THAN 10

From March 1967 Grade Tchr.

Use Calendar for month in  
Base 7 numerals

						1
2	3	4	5	6	10	11
12	13	14	15	16	20	21
22	23	24	25	26	30	311
32	33	34	35	36	40	41
42	43					

STUDENT WILL BE ABLE TO IDENTIFY ROMAN NUMERAL  
MULTIPLICATIVE BAR, REPETEND BAR, RADICAL SIGN,  
SUBSET AND EMPTY SET SYMBOLS, IS EQUIVALENT TO,  
AND IS APPROXIMATELY EQUAL TO, NOTATION

### III. Symbols

7 S

1. Given a list of 10 symbols, the student will correctly match the symbol to the definition with 90% accuracy.

Readiness: Familiarity with basic set notation and with Roman numerals.

HRW 7,

pp. 9-11, multiplicative bar  
265-271, square root or radical sign  
138-140, subset  
116-117, empty set  
166, is equivalent to  
207-210, bar for repeating decimal  
(repetend)  
268-269, is approximately equal to.

REINFORCE FRACTIONS, RATIONAL NUMBERS, & RATIO

### III. Fractions

7-8 REMEDIAL

1. Given a set of indicated operations on fractions, rational numbers, and ratios, the student can solve with 60% accuracy.

Readiness: The learner can recognize fractions and ratios. He can see the fraction as a ratio or indicating division.

AW 1651 - pp. 260-293; 294-326

AW 1661 - pp. 236-265; 268-285; 288-317;  
318-331.

AW 1671 - pp. 188-201; 204-229; 232-271;  
274-281.

Cyclo-Teacher M67, M78.

Transparencies

Webster, Osborn: EXPLORING ARITHMETIC  
c 1962, pp. 12-13; 17; 21; 23; 301-306.

HRW, Bernstein & Wells: TROUBLESHOOTING  
MATHEMATICS SKILLS c 1969 Chapt.3(p.104)

Make "Frac" game - uses sequences starting with lowest fraction. Student must recognize and arrange from lowest up.

# RETEACH DECIMAL FRACTIONS

## III. Decimals

### 7-8 REMEDIAL

1. Given a list of indicated operations involving decimals, the student will perform the indicated operations with 60% accuracy.

Readiness: Place value concept; familiarity with common fractions.

AW 1651 - none  
AW 1661 - pp. 332-343  
AW 1671 - pp. 284-317

Filmstrips

Stress re-writing decimal divisor problems instead of moving decimal point.

Houghton-Mifflin, Dolciani et al: MODERN SCHOOL MATHEMATICS 8, c 1967, Chapt. 9, p. 223.

Webster, Osborn et al: EXPLORING ARITHMETIC 8, c 1962, p. 61

"Deco" Game - p. 15, Ginn's Games  
"Decimal Relay," - p. 15, Ginn's Games

# RETEACH CONVERTING DECIMAL FRACTIONS TO PER CENTS AND RETEACHING OPERATIONS WITH PERCENTS

## III. PER CENT

### 7-8 REMEDIAL

1. Given a set of ten numerals, the student can convert, with 60% accuracy, to the other two equivalent forms (decimal-fraction-percent).
2. Given a list of indicated operations involving percent, the student can accurately solve a minimum of 50%.

Readiness: Working knowledge of decimal fractions.

AW 1651 - none  
AW 1661 - pp. 344-351  
AW 1671 - pp. 320-337

Allyn & Bacon, Stein: FUNDAMENTALS OF MATHEMATICS c 1968, pp. 149-171.

HRW, Nichols: MODERN ELEMENTARY ALGEBRA c 1965, pp. 223-224.

HRW, Bernstein et al: TROUBLESHOOTING MATH. SKILLS c 1969, Chapt. 9, p. 253 et al.

"Over Orange" - Rummy style game-Ginn's Games, p. 13.



## REINFORCE MULTIPLICATION & DIVISION

### III. Multi.-Division

#### 7-8 REMEDIAL

1. Given a list of ten multiplication and division problems, the student will solve 7 out of the 10 correctly.

Readiness: Addition and subtraction facts are used with reasonable accuracy. Student is aware that multiplication and division are inverse operations.

AW 1651- pp. 82-111; 114-133; 176-205; 208-243.

AW 1661 - pp. 64-95; 122-137; 152-175; 176-215; 288-317.

AW 1671 - pp. 36-49; 232-271; 88-91; 100-123.

Cyclo-Teacher,  
H 22-31 on Multiplication  
H 32-44 on Division

Time Tests

HMW, Bernstein et al, TROUBLESHOOTING MATH. SKILLS, pp. 57-103; 165-192.

## REVIEWING ESTIMATION

### III. Estimation

#### 7-8 REMEDIAL

1. Given a set of ten indicated operations, the student will estimate 60% of the answers within 25% accuracy.

Readiness: Understanding of rounding and place value.

AW 1651, pp. 138-149

AW 1671, pp. 70-79

Recipes (to estimate quantities to have on hand)

Drivers Tests - estimating distances, speed, time

Checking grocery size packages for estimating best buy.

REVIEW OPERATIONS ON INTEGERS WITH EXCEPTION OF  
DIVISION

III. INTEGERS

8 REMEDIAL

1. Given a set of operations (addition, subtraction, or multiplication) to perform upon some integers, with 60% accuracy the student will solve the problems.

Readiness: Familiarity with the numberline and basic operations.

AW 1671 - pp. 342-352

Transparencies

Film Strips

Cyclo-Teacher - M95,96,97,100,101,102, 103, 1104, 105, 106.

Shuffleboard with negative numbers.(use masking tape on floor)

REINFORCE ADDITION AND SUBTRACTION

III. Operations

7-8 REMEDIAL

1. Given a list of ten addition and subtraction problems, the student will compute with 70% accuracy-- i.e., solve 7 out of 10 problems.

Readiness: Student sees addition and subtraction as inverse operations, and knows one-digit number combinations.

AW 1651 - pp. 20-47;50-81(1-,2-,& 3-digits)  
AW 1661 - pp. 28-61;268-287 (4-digit no. and regrouping)  
AW 1671 - pp. 30-35; 204-229(rationals);  
82-87; 92-93; 95-99; 124-127.

Number Line.

Function Machines

Magic Squares

Cyclo-Teacher M 4-12 (Addition Facts)

M 13-21 (Subtraction Facts)

Time Tests

HRW, Bernstein et al: TROUBLESHOOTING MATHEMATICS SKILLS, c 1969,  
p. 4-56 Practice Sets  
p. 132-164 - Rational Numbers

STUDENT WILL BE ABLE TO RECOGNIZE A NON-  
TERMINATING DECIMAL

III. Irrationals

7 S

1. Given two numbers to compare by division, the student can correctly identify (in writing) a non-terminating decimal.

Readiness: Student can divide with reasonable accuracy; student must "see" fraction as a comparison of two numbers by division.

HRW 7, pp. 217-218; 265-266; 202-204;  
206-217; 265-266.

Contrast repeating and non-repeating decimals.

THE STUDENT IS INTRODUCED TO THE CONCEPT OF  
DIRECTED WHOLE NUMBERS BY MEANS OF ORDERED  
PAIRS; HE LEARNS TO ADD AND SUBTRACT WITH  
THESE INTEGERS.

III. Integers

7 S

1. The student will be able to match 5 out of 6 given integers with an equivalent ordered pair.
2. The student will correctly compute 8 out of 10 addition or subtraction problems (including word problems) involving integers.

Readiness: Student has working knowledge of whole numbers and is able to add and subtract whole numbers with reasonable accuracy.

HRW 7, pp. 168-180

Student learns that an integer may be represented by any one of an infinite set of equivalent ordered pairs.

For addition, student learns to add first terms of ordered pairs together and second terms together and then to express answer as integer.

In subtraction the student is made aware of choosing big enough equivalent pair for first term in subtraction problem and then subtracts respective members of second expression from the members of the first expression, giving the answer in integer form.

STUDENT WILL BECOME FAMILIAR WITH FUNDAMENTAL OPERATIONS INVOLVING DIRECTED NUMBERS (REVIEW ADDITION-SUBTRACTION; ADD MULTI. - DIVISION )

### III. Integers

8 S

1. Given a list of 25 problems in various fundamental operations involving directed numbers, student will correctly solve 20 out of 25.

Readiness: Student is acquainted with concepts of directed numbers.

HRW 8, pp. 193-234 and 235-263.

Student learns meaning of additive inverse.

Student can see subtraction as process of adding additive inverse.

Student learns 0 as sole integer which is its own additive integer.

Directed number = real number = directed real number in this text.

Student practices comparing directed numbers.

Student develops pattern for multiplication of signed numbers.

Division is shown to be inverse operation of multiplication and that this works for directed numbers, using terms of "reciprocal" = "multiplicative inverse."

STUDENT LEARNS TO DIFFERENTIATE BETWEEN "SQUARE" AND "SQUARE ROOT, AND TO FIND APPROXIMATE SQUARES OR SQUARE ROOTS.

### III. Squares & Square Roots

7 S

1. By using the tables of squares and square roots in his textbook, the student can select the correct square or square root, as indicated, of 4 out of 5 given numbers.

HRW 7, pp. 265-299

Student is introduced to p. 420 of text showing number facts. Cube roots are introduced incidentally.

Readiness: Knowledge of exponents of factors.

III. REAL NUMBER OPERATIONS  
SUPPLEMENTARY ACTIVITIES

page 1

Kenworthy Educational Service, Inc.  
Buffalo, N.Y.  
SELF TEACHER, TESTER, CHECKER, c 1963  
Tagboard with 100 combinations with  
cut-out rectangular spaces below  
every 4 problems for write-in answers.  
Answers shown on reverse side.  
Code no. is No. 2700. Price is  
about \$3.00 for a dozen cards.

ADDITION RELAYS (could also be used  
for multiplication and subtraction)

4-digit numbers are placed on board  
(or overhead?). Class is divided  
into 4-member teams. Each member  
adds one column. First team to get  
correct answer wins.

ADDITION GAMES

Commercial games such as dominoes,  
cribbage, parcheesi, yahtzi.

ADDITION-SUBTRACTION QUIZMO

Milton Bradley  
#58480-049 \$2.00

COMPUTATION -

A punch-card "adding machine" your  
pupils can build.  
Math. Teacher Vol. LII, No. 6,  
Oct. 1959, p. 471.

AN EXHIBIT ON ESTIMATION

Math. Teacher Vol. LIII, No. 5,  
May, 1960, p. 388

DECIMALS AND FRACTIONS IN MEASUREMENT  
World Book Co., GROWTH IN ARITHMETIC  
8, Revised, Teacher's Edition, c 1957,  
pp. 51-59.

Fractions and decimals in the metric  
system; practical problems in use of  
fractions; practical problems using  
decimals. (Can involve "tolerance.")  
This book is also good on fractions  
and decimals, including word prob-  
lems, in earlier parts of this book.

FRACTIONS (Practice)

Webster, Osborn et al, EXPLORING ARITH.8:  
Addition: 10-13, 302, 306  
Change to decimals - 29  
Change to percent - 48  
Division: 16-17, 305-306  
Multiplication: 14-15, 101, 135, 304, 306  
Practice: 11-17; 101; 133; 135; 139; 159;  
211; 247; 271; 281; 301; 307; 314.  
Subtraction: 10-13; 303; 306.  
Problems: 8-12, 14-17; 20-21; 56; 67.

FRACTION QUIZMO

Milton Bradley  
#58480 \$2.00  
Suggest for 7th-8th Remedial using  
plastic covers for cards which can then  
be marked and erased, rather than the  
tiny cardboard buttons supplied.

FRACTIONS - "Save Those Egg Cartons"  
Arith. Teacher Vol. 14, No. 7, Nov. 1967,  
p. 578

FILM LOOP - FRACTIONS

Encyclopedia Britannica  
Super 8 MM Color No. S-80075 \$17.60  
or  
8 MM Color No. R-80075 \$16.00  
Fractions on the number line -- only  
like fractions can be added or sub-  
tracted.

SIGNED OR DIRECTED NUMBERS,  
FILM LOOP - DIRECTED NUMBERS

Encyclopedia Britannica  
Super 8 MM Color No. S-80076 \$17.60  
or  
8 MM Color No. R-80076 \$16.00  
Defines signed numbers by numberline and  
clarifies nature of addition and subtrac-  
tion.

FILM LOOP - SIGNED NUMBERS

Encyclopedia Britannica  
Super 8 MM Color No. S-80083 \$17.60  
or  
8 MM Color No. R-80083 \$16.00  
3 sets of integers -- shows relationship  
between positive and negative integers by  
utilizing as holes cut from fabric.

### III. REAL NUMBER OPERATIONS

-- page 2

#### RATIO AND PROPORTION

Webster, Osborn et al, EXPLORING

ARITH. 8:

Ratio: pp. 18, 50, 219, 2-3, 226-227

Problems: 18-19; 50-51; 219-220; 226-227;

230-231

Proportion: 221-224; 226-230

#### MULTIPLICATION-DIVISION QUIZMO

Milton Bradley # 58480-050 \$2.00

#### MULTIPLICATION DRILL -

Game - "The Winning Touch" - type of Scrabble with multiplication facts.

No. 702

"Winning Touch"

Ideal School Supply Co.

Oak Lawn, Illinois 60453.

Think this would work well in remedial 7th and 8th in place of regular drill.

of one denomination, that is, all in quarters, or all in five-dollar bills, etc. Let's see how many twenties, tens, fives, ones, halves, quarters, dimes (tenths of a dollar), nickels (twentieth of a dollar) or pennies (hundredths of a dollar) I could get at a time:

$$\begin{array}{ll} 20 \div 20 = 1 & 20 \div 1 = 20 \\ 20 \div 10 = 2 & 20 \div 1/2 = 40 \\ 20 \div 5 = 4 & 20 \div 1/4 = 80 \end{array}$$

$$20 \div 1/10 = 200$$

$$20 \div 1/20 = 400$$

$$20 \div 1/100 = 2000$$

#### USE EITHER WITH DIVISION BY PROPER FRACTION OR AS MONEY EXERCISE:

Why is the answer bigger than the number you had at first when you divide by a proper fraction? Actually, the question in most people's minds, although they do not say so, is, "Since you do get a quotient larger than the dividend when you divide by a number less than 1, why in the name of reason do you call it division?" Here are two activities:

1. Talk about mathematical principles.

$$20 \div 4 = 5 \text{ because } 5 \times 4 = 20;$$

and thus, by same pattern

$$20 \div 1/4 = 80 \text{ since } 80 \times 1/4 = 20.$$

2. Others still say, "Yes, but..." For these, the following story is the most productive I have yet found:

I am going to the bank to cash a check for \$20. I decide to get my \$20 in coins or bills all

EXAMPLE-WITH-A-HOLE-IN-IT by Jo Phillips, Feb. 1968 Instructor, p. 39.

Put on the board an example-with-a-hole-in-it such as

$$\begin{array}{r} 482 \\ \times 36 \\ \hline 2892 \\ \hline 17352 \end{array}$$

"Tell pupils to imagine that for a few horrible minutes they have completely forgotten all the multiplication facts. Nevertheless, using just what they see here, they can find the answer to  $482 \times 30$ . If they sense immediately how to do this (17352-2892), they understand the multiplication algorithm, whether or not they usually write the '0' in the second partial product.

SUPPLEMENTARY ACTIVITIES

**SYMBOLS** - "We've Been Framed" by Dr. Jo Phillips, Oct. 1967 Instructor, p. 103.

"Open sentence" means exactly what the words suggest - An open sentence has one or more holes in it. Indicate "hole" by writing a squiggle other than a number symbol or operation symbol or relation symbol. Call this squiggle "variable." Variables do not ask question, do not hold place for a specific numeral or a specific symbol - only indicates place where hole occurs. Open sentence is neither true nor false; could be either depending on replacement. Set of all values of a variable is called its "domain."

When we show what somethings IS NOT, we reinforce what it IS, as well as open the door to broader concepts. Instructional program in variables should go farther than solution sets of open sentences.

$5 + ? = 7$  Questionmark is not a variable but shorthand for "what number?"

**SYMBOL RECOGNITION GAME** (by rk)  
Similar to Fish.  
Could be used as a team game with any two correct acceptable.

Three sets of cards can be made--one card showing the actual symbol itself, a second card with the name of the symbol, and a third with a brief description or other clue as to how the symbol is used.  
Winner is the one able to collect the most 3-card sets.

**SQUARE ROOTS AND SQUARE ROOT TABLE**  
Harcourt, Brace & World, Court:  
**GROWTH IN ARITHMETIC 8**, Discovery Edition, p. 285

**PER CENT**

Webster, Osborn et al, **EXPLORING ARITH.8**, c 1962.

Unit 3, pp. 47-70

Interchanging decimals and fractions and percent, pp. 47- top quarter p. 51.

Word problems, pp. 51-53; 54

Finding % of a number: p. 52, 9-18,, p. 54 (1-4)

Finding what % a part is of a whole, p. 53 (9-16)

Finding less than 1% - p. 54

% of increase-decrease - p. 55

Finding whole from part - p. 56

Finding part from % or whole - p. 58  
word problems (we do not use rate-base wording)

Finding % from part and base - word problems and numerical problems - p. 59.

Word problems with fractions or percents- p. 67

Good check-up tests on pp. 68, 69, 70.  
Also Suppl. Test, p. 70A.

**PER CENT.** Amer.Book Co., Deans et al, **STRUCTURING MATHEMATICS**, p. 229 et al, Chapter 8.

This book has good portions (of this chapter) for equivalent fraction-decimal-percent practice and much good practice work. We at our level have not been emphasizing "rate" and "base" in percentage, feeling that these two words create confusion, and this chapter does utilize these terms in the standard tradition. However, the Shortcuts, Problems, review on p. 255 and Chapter Test on p. 256, are very good. This chapter brings in simple interest and compound interest if desired.

**PER CENT** - World Book Co., **GROWTH IN ARITH. 8**, Revised, Teacher's Edition, c. 1957  
p. 259 - Word problems concerning cost, selling price, and per cent.  
p. 260-261 Word problems involving interest, percent, promissory notes. etc.



### III. REAL NUMBER OPERATION

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#### TRANSMOBILE (Abacus for Overhead)

Transmobile No. TM-2  
Weber Costello, Chicago  
(developed by Instructional  
Dynamics Inc.)

Use for addition, other bases, place value, subtraction, fractional parts, possibly multiplication (not division--too difficult), reading and writing of numbers, work with decimals.

#### PRINCIPLE OF ORDER OF OPERATION

Singer/Random House, MODERN MATH.,  
c. 1965, 1961 (Paperback)  
Included under discussion on Principles (closure, commutative, etc.)

DO-IT-YOURSELF CHECK CHART by Ken Baird, Grade Tchr. Jan 1968, p. 78

Since the four basic skills are still the bulk of a child's math education, this teacher has made a classroom feature of the 29 basic computational problems which are generally considered essential in 4th through 7th grades. Although the number of problems is infinite, they all fit into a category of 29 problems. Each problem is put on a card, numbered, and mounted in order across the front of the room. Thus the student can see: the whole pattern of skills to be learned

the size of the learning task  
how far they have progressed  
how much farther to go.

Keep chart for each section on which child's progress through the 29 is recorded. Acts both as record and stimulus. Lets each pupil know where he stands in relation to others in his class (or in his school). Parents like to see something this concrete. This work is independent of regular classwork lessons. It is voluntary and does not affect regular grades. Work on charts is done in spare time. Problems must be done in sequence. Teacher looks at problem student has done and gives him one similar in category. If he can handle it, his line on the chart

moves ahead one more space. Cards help student to pinpoint his problem so that exact help can be given.

$$\begin{array}{r} 1) \quad 72 \\ \quad 31 \\ \hline \end{array} \quad \begin{array}{r} 2) \quad 42 \\ \quad -31 \\ \hline \end{array} \quad \begin{array}{r} 3) \quad 4002 \\ \quad -3999 \\ \hline \end{array} \quad \begin{array}{r} 4) \quad 24 \\ \quad \quad 2 \\ \hline \end{array}$$

$$\begin{array}{r} 5) \quad 48 \\ \quad x31 \\ \hline \end{array} \quad \begin{array}{r} 6) \quad 347 \\ \quad x47 \\ \hline \end{array} \quad 7) \quad 7/25$$

$$8) \quad 7/2461 \quad 9) \quad 54/7471 \quad 10) \quad \begin{array}{r} 4/8 \\ -1/8 \\ \hline \end{array}$$

$$11) \quad \begin{array}{r} 3/8 \\ + 1/4 \\ \hline \end{array} \quad 12) \quad \begin{array}{r} 1/7 \\ + 2/7 \\ \hline \end{array} \quad 13) \quad \begin{array}{r} 3 \\ + 1/8 \\ \hline \end{array}$$

$$14) \quad \begin{array}{r} 3 \frac{1}{3} \\ - 2 \frac{5}{7} \\ \hline \end{array} \quad 15) \quad \begin{array}{r} 3 \frac{1}{2} \\ + 2 \frac{1}{4} \\ \hline \end{array} \quad 16) \quad \begin{array}{r} 7 \\ - 3/5 \\ \hline \end{array}$$

$$17) \quad 2/7 \times 1/5 \quad 18) \quad 1 \frac{3}{5} \times 7 \frac{1}{4}$$

$$19) \quad 4 \times 1 \frac{1}{7} \quad 20) \quad 1/3 + 1/7$$

$$21) \quad 1 \frac{1}{3} + 1 \frac{1}{5} \quad 22) \quad 7 \frac{1}{7} + 7$$

$$23) \quad \begin{array}{r} 12.4 \\ + 3.7 \\ \hline \end{array} \quad 24) \quad \begin{array}{r} 4.7 \\ - 1.4 \\ \hline \end{array} \quad 25) \quad \begin{array}{r} 10.3 \\ \quad x 4.4 \\ \hline \end{array}$$

$$26) \quad 23/\overline{74.5} \quad 27) \quad 3.1/\overline{17.4}$$

$$28) \quad \begin{array}{r} .7 \\ x .2 \\ \hline \end{array} \quad 29) \quad \begin{array}{r} 74.3 \\ x 2.3 \\ \hline \end{array}$$

#### TRACHTENBERG SPEED SYSTEM OF BASIC MATH.

Translated and adapted by Ann Cutler and Rudolph McShane. c. 1960 \$4.95

Doubleday & Company, Inc.

Garden City, N. Y.

c 1960 by Ann Cutler

Library of Congress Card 60-13513

Speed systems for multiplying by 11, 12, etc., rapid addition, division, square roots, etc.



SUPPLEMENTARY ACTIVITIES

COMMISSION, DISCOUNT, INSURANCE,  
BANKING

Webster Publ. Co., Osborn et al, EX-  
PLORING ARITH. 8 :

Commission: pp. 62, 123

Discount: pp. 78, 65, 64

Insurance: pp. 106-107; 98-100; 105;  
96-97; 102-105;

insurance problems: 95-100; 102-  
107.

Banking: 71-73; 76-78; 80-81; 83-85;  
88-89.

MATH AIDS YOU CAN MAKE, by Dr. Jo  
Phillips, Oct. 1966 Instructor,  
p. 113.

"Old Maid" type-unlined filing cards  
and marking pen - all kinds of  
drills.

21 cards - 3 players - 1 unmatched  
card.

(10 cards with number combinations  
and 10 with answers)

Players find matching pairs by  
drawing from one another's hands  
as in "Old Maid." Each group  
should contain at least one pupil  
sure enough of the facts to check  
on the rest. Decide how to score  
the first time the game is played.  
Frequently pupils will suggest  
specific variations..."Why didn't  
you make the cards so that..."

If older pupils make the cards  
for younger ones, they should be  
sure to try them out to eliminate  
possible "bugs" that will confuse  
the younger ones.

"Hundreds Chart" may be made on  
poster paper, but more durably on  
wood, 10" x 10". Use cuphooks or  
L-shaped screws to place in each  
square. Numbered key tags may be  
hung on the hooks in any way the  
occasion may warrant. Device can be  
used for all kinds of exercises, from

finding missing numbers in a sequence to  
sifting out prime and composites, etc.  
Some tags can be printed with operation  
signs and the chart used in working with  
open number sentences. May also be used  
like a geoboard with rubber bands.

PEGBOARD may be made from tempered masonite  
pegboard or even scraps of soundproofing  
material. Golf tees and tapered pieces  
of doweling make excellent pegs. Every  
teacher knows at least 5 ways to use a  
pegboard in math. class.

CYCLO-TEACHER LEARNING AID

World Book Co. (Field Enterprise)

Circular several-windowed device to en-  
able student to turn crank to ask question  
give space for his written answer to  
printed question, and next crank reveals  
correct answer for immediate checking.  
Sets of discs for Cyclo-Teacher include  
Social Studies, Mathematics, etc. See  
Supplementary List of Materials and  
Equipment for more detailed prices.  
(\$49 for 2 complete sets.)

"CALCULATOR'S CUNNING, The Art of Quick  
Reckoning," by Karl Menninger.

M-12 \$4.50

Cuisenaire Co. of America, Inc.

12 Church St.

New Rochelle, New York 10805

CLASSROOM COMPUTER

Math. Teacher LIX, No. 4, April 1966,  
p. 356

Idea of using gallonage computerheads  
from discarded gasoline pumps for class-  
room computers by putting handles on--  
see picture in above article.

### III. REAL NUMBER OPERATIONS

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#### SUPPLEMENTARY ACTIVITIES

##### MATHEMATICS FIELD TRIPS

Arithmetic Teacher Vol. 15, No. 4,  
April 1968, p. 332 and 340.

Suggested:

Grocery Store  
Hardware Store  
Feed Store  
?Equipment Store

##### CONTINENTAL PRESS MASTERS

Packets of Masters for Transitional  
Math, Learning New Concepts, and  
Modern Math for Jr. High. For Liquid  
Duplicator. Teacher's Key free.

Grade 7: Learning New Skills in  
Arith., Parts 1 & 2  
Jr. High Arithmetic, Sem-  
esters 1 & 2

Grade 8: Learning New Skills in  
Arith., Parts 1 and 2.  
Jr. High Arith., Semester  
1 and 2

Ungraded Jr. High: Modern Math for  
Jr. High  
Working with Decimals (6+),  
Working with Percentage (7-9)

\$4 per packet

Continental Press

Elizabethtown, Pennsylvania 17022

or

Elgin, Illinois 60120

NUMBER LINE, by Dr. Jo Phillips, Oct. 1967  
Instructor, p. 119

Counting forward - readiness for addition  
Counting backward-readiness for subtraction  
Skip counting - relate to multiplication  
Skip counting backward - relate to division.

Counting forward and backward serves as  
simplest possible way to introduce negative  
numbers. Start with number line on which  
only positive whole numbers are labeled.  
Make up instructions such as, "Start at 3;  
go 2 forward and 7 backward; where do you  
land?" Let the children name the landing  
places. Can also then introduce the idea  
of opposites using two frogs who live at 0  
and who never want to go the same direction.  
Only time together is at home.

Dr. Phillips suggests using three dots to  
indicate infinite line extension and re-  
serving headed arrows for specific dis-  
placements.

Many kinds of "Track" games are useful here,  
including "wild variations of  
Parcheesi and Uncle Wiggly."  
One or two dice or cards can be  
used to determine size of move.  
Can be as simple or complicated  
as desired. (Can use moves  
which are multiples of 3, etc.,  
or number which is congruent  
on a round number of "clock"  
line to 3 modulo 8...)

##### S.R.A. TAPES

1 Kit of Computer Skills 3-3350 \$ \_\_\_\_\_  
1 set #2110370 plus \$450  
4 Cassette recorders \_\_\_\_\_

Ordered by Central Jr. High May '69.

### III. REAL NUMBER OPERATIONS

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#### GAMES, PUZZLES, FUN THINGS

#### 2 FUN PROBLEMS

Webster, Osborn et al, EXPLORING ARITHMETIC 8, p. 70 A

DOMINO GAMES, by Jo Phillips, Dec. 1967 Instructor, p. 108

Commercial dominoes up to double 6 or double 9, per maturity level, may be used, first just matching dots, next laying doubles crosswise, and then allowing branching from the doubles, and standard rules. These can be used for addition skills and identifying multiples of 5.

More extensive use can be made of dominoes from cardboard, plastic, or wooden sticks such as those used in windowshades. These can have number combinations, either addition or subtraction, on one side of domino:

$$\begin{array}{ccccccc} 6 & + & 3 & & 4 & & 9 & - & 2 & & 7 \\ & & & & & & & & & & \end{array}$$

The second domino is a double. Such dominoes can be made as simple or as difficult as you wish. Often pupils may make the games either for their classmates or for other classes. Usually they are tried out at home and the whole family gets interested.

MAKE A SCENE, by Carol Cowles in Nov. 1967 Instructor

To give more interest to math drill, I often make up some duplicated sheets of problems, enclosing each problem in an area. When the problems are done, students color the areas according to a key I set up-- areas with answers of 12, blue; answers of 10, red; etc. If problems have been done correctly, colored areas make a picture.

IT'S IN THE BAG, by Jan Cummings in Nov. 1967 Instructor, p. 114

When students seem to need a change, we have "It's in the Bag" Day. Activities are written on separate pieces of paper and put into a small bag attached to chalkboard(?) and labeled "It's in the Bag." When we are ready for a new activity, a child pulls out a paper. No one knows what we'll do until the paper is read.

We think this could be adapted to a Math Laboratory, etc., or perhaps to the sequence of items in a daily class lesson.

YES, MATH CAN BE FUN! Book by Louis Grant Brandes, c 1960.

263 pages of problems, puzzles, math magic, games, and other math entertainment. Includes biographies of great mathematicians.

J. Weston Walch, Publisher  
Box 1075  
Portland, Maine

GINN GAMES FOR ARITHMETIC pamphlet  
26-3, 59

Ginn & Company  
Arlington Heights, Illinois 60005

This is an excellent booklet containing many games, and adaptations, which are very usable.

PUZZLES - A CROSS NUMBER PUZZLE

Math. Teacher Vol. L, No. 8, Dec. 1957,  
p. 568

TUF Number Game

TUF  
P.O. Box 173  
Rowayton, Conn. 06853 (brochure)

Also listed in Guisenaire.

III. REAL NUMBER OPERATIONS  
 --- page 8  
 GAMES, PUZZLES, FUN THINGS

MAGIC SQUARE FOR THE NEW YEAR  
 Math Teacher Vol. LXI, No. 1, Jan.  
 1968, page 18

Reference:  
 Simon & Schuster, New York  
 THE SCIENTIFIC AMERICAN BOOK OF  
 MATHEMATICAL PUZZLES AND  
 DIVERSIONS, Chapter 11.  
 by Martin Gardner

CALENDAR ARITHMETIC  
 Arith. Teacher, Vol. 16, No. 1, Jan.  
 1969, page 69

1. 3 successive dates in same month

sum divided by 3 = middle date;  
 add 1 and subtract 1 for other  
 2 dates.

2. 3 successive dates in same column (same day of week)

Sum + 3 = middle date, add 7 and  
 subtract 7 for other 2.

3. Four consecutive dates

$\frac{\text{Sum} - 6}{4}$  = smallest date; add 1,  
 2, 3  
 for other 3.

4. Four successive dates in same column:

$\frac{\text{sum} - 42}{4}$  = smallest date.  
 Add 7, 14, 21 for other three.

5. Four dates which form a 2x2 square:

$\frac{\text{Sum} - 16}{4}$  = smallest number.

Add 1, 7, 8 for other 3.

(continued in next column)

6. Eight dates which form a 2x4 rectangle:

$\frac{\text{Sum} - 40}{8}$  = smallest number.

Add 1, 2, 3, 7, 8, 9, 10 for others.

TRACK GAMES = NUMBER LINE GAMES, by Jo  
 Phillips, 1967 Dec. Instructor, p. 89.

Try first on inexpensive materials - can be  
 converted into more durable version.

Basically, need starting place, track to be  
 traversed in accordance with rules and pro-  
 cedures (which vary with variations) and a  
 spot a player must reach to win. Distin-  
 guishable markers are needed for each player  
 (colored disks or buttons, or different tiny  
 toys) and dice, cards, or spinner to indi-  
 cate the moves.

Use number line with "dark cave" indicated  
 on spot 7, etc., for example. Player who  
 lands on already occupied space suffers some  
 kind of penalty (stops at nearest unoccupied  
 space behind that one) or a bonus (goes a-  
 head to nearest unoccupied space ahead of  
 it). For complications, may throw 2 dice or  
 pick up 2 cards, etc., or some cards may  
 have special instructions "Go to Dark Cave,"  
 etc. (Game should reveal quick addition, etc.,  
 or those ready to go on to more complicated  
 games.)

Harder: Tracks may go up to 50 or 100 in-  
 stead of original 30, or use steps as multi-  
 ples; marking steps by other than single  
 units may require rounding the numbers they  
 draw to the nearest multiple. Cards better  
 than dice for upper grades; with dice use  
 stated penalty for certain throws like  
 double ones and bonus for double five, etc.  
 Try to have one pupil in each group who is  
 likely to get directions straight and who  
 is confident of required number facts.

#### PROBLEM CARDS (Domino Style)

Two-part cards are prepared (maybe 4x2"), one-half giving a number fact and the other half an answer to some other problem. The first player puts down a card on which the second player can play on either end (supplying the number fact to the answer given on the first card or else the answer to match the number fact given on the first card). Each player starts with 4 or 5 cards and draws a card from the pack at the completion of each play. The first player to get rid of all cards in his hand is the winner.

#### TAKE IT OR LEAVE IT (Review) Tic-tac-toe type.

Questions are prepared on individual cards and placed in a box. Class is divided into two teams. Teacher selects random card and briefly describes question ("This is about division") asking team Do You Take It or Leave It? If the card is turned down, a second card is drawn which the team MUST accept and the rejected card is returned to the box. Questions are directed at teams alternately. If an answer is incorrect, the other team gets the opportunity to answer with double points awarded for correct answer. Team with correct answer gets to mark its X or O in tic-tac-toe box. Any three marks in column, row, or diagonal means a winner.

#### MYSTERY SIGN (Drill on Definitions, etc.)

One student is chosen to be "It." The definition or symbol is written on a card or paper and pinned to "It's" back so that he is unable to see it. He must discover by questioning students, with only Yes or No answers allowed, what is his mystery sign.

#### SECRET AGENT REVIEW

Students are assigned the ordered pair which represents their seat number. Each student prepares 5 questions for review. Teacher selects first student who poses his question and decides what number Secret Agent should answer it. If he answers correctly, that Secret Agent has his turn to pose his question and assign the answer. If an incorrect reply is received, the questioner may ask for volunteers and the correct answerer becomes the questioner.

#### MATHEMATICAL TERM BINGO

Needed: 2 pegboards, colored markers, 40 words on cardboards for pegboard.

From a pegboard list of 40 words prepared by the teacher, students may select any 16 for his squares formed by folding his paper. Once he has written his chosen 16 words in his squares, the game can begin. Student selected to go to the board chooses a word from the pegboard, pronounces it and moves it to the smaller pegboard. Student who has this word on his paper covers it with colored marker (oaktag square?). Student at board chooses another student to select the next word. Winner is student who completes row, column, or diagonal first, providing he can pronounce (or define?) all the words.

INTEREST GETTERS, by Karl G. Zahn,  
Arith. Teacher, April 1968, pp. 372-374

3 "problems" for maintaining pupil interest:

- 1) Number pyramid of multiplications
- 2) Pat and Mike, "7 tons of coal at \$10 a ton is \$49."
- 3) Arab and his three sons and division of 17 horses.

### NUMBER BASEBALL

Class is divided into two teams. Review questions gain a "single" for each correct answer and an "out" for each incorrect reply. Teams are questioned alternately. Three "outs" constitute an inning. More difficult questions can be labeled "double," "triple," or "homer."

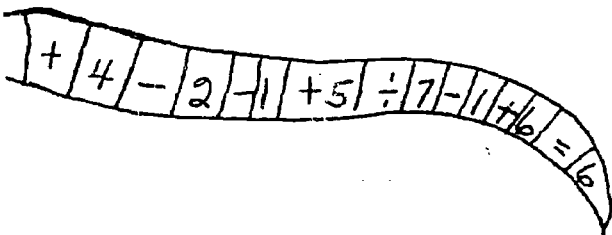
Questions can be selected at random by drawing from envelope. Each captain keeps score for his team. Variations are numerous. Students can compile questions for random selection.

This technique could also be used for drills.

NUMBER WORMS, by Alfred and Patricia Lazar, Nov. 1967 Instructor, p. 144

Number worms are long segmented drawings with a computation to be done in each segment. They make excellent warm-up activity or for any few spare minutes. Worms can vary in size and range of operations involved and provide high motivation.

(We suggest making them on plastic--possibly discarded overhead projector strips--so they can be used on overhead, possibly in colors, and thus eliminate having to duplicate individually.)



OPERATION BOTTLECAP, BY Marjorie Bacon, p. 466 Arithmetic Teacher Oct. 1965

Collection of bottle caps can be sprayed red or some such bright color, and used for counters, to illustrate operations, and to illustrate properties such as one-more-than or one-less-than...

Author seemed to think that the "project" type of collection added considerably to the success of Operation Bottlecap.

She suggests using these counters flat side down and fluted side up so that desks will remain unscratched and yet these counters are easily fingertip manipulated in this position.

20 counters per student is not enough for the "one-more-than" relationships through the number 8.

OPTICAL ILLUSIONS - "April Fool Math" by Dr. Jo Phillips, April 1968 Instructor

1. Use 2 segments of identical length to make an upside-down capital T. At top write, "Which segment is longer?" and at bottom, "How can you be sure?" Probably best to use same color for both segments unless you want to display effect of color on length.
2. Cut out two circles, same size, one from black and one from white. Mount on opposite color background so that background is at least twice as wide as circle. Display side by side with sign at top, "Which circle is larger?" and at the bottom, "How can you be sure?" (Does it make any difference if the figure is a different shape than a circle -- square, triangle, etc.??? Let students experiment.)



### III. REAL NUMBER OPERATIONS

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GAMES, PUZZLES, FUN THINGS

#### OPTICAL ILLUSIONS (continued)

3. Make drawing like example and ask whether any segment on the left belongs to the same straight line as the segment on the right. As usual, ask how to verify the answer.



Optical illusions are suitable for all grade levels and these illustrate the three general types of illusion.

MOEBIUS or Möbius STRIP, from April Fool Math by Dr. Jo Phillips in April 1968 Instructor, p. 88

Have fun with a Moebius strip, a solid with just one surface. Cut a strip of paper, not stiff, about two inches wide and 18 inches long. Newsprint or grocery bag is fine. Take a strip, give a half twist, overlap the ends, and paste them together (like a belt twisted just enough to go upside down through the buckle). Let the past dry. Then think of this "loop" as a highway. An ant is walking along the center line. Draw the ant's path, all the way. Surprised? Cut along the line you just drew. Gee Whiz! Repeat the same thing with the ant walking along this new highway. Cut again. Can you visualize a bulletin board display of the stages of this activity? It shows one thing which may happen in a non-Euclidian space. It interests people of all ages.

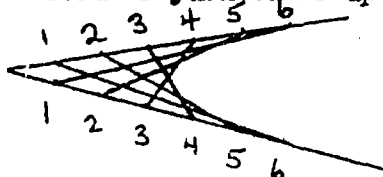
#### FUN WITH NUMBERS (for 6th grade up)

Let your class try this. Convert  $1/27$  to the decimal form. Then do  $1/37$ . Now, try  $1/81$ . Exasperating! ( $1/27 = .037037$ ;  $1/37 = .027027$ ;  $1/81 = .0123456...$ )

CURVE STITCHING, from April Fool Math by Dr. Jo Phillips, April 1968 Instructor, p. 88

Draw any angle on a piece of plain paper. Mark each side off in units of a convenient size (perhaps quarter inches). Number the marks starting from the vertex of the angle so that you have the SAME NUMBER on each side. Now, with a needle and thread, or a ruler and pencil, connect the last mark on one side of the angle with the first mark on the other side. Then go down a notch on the first side and up a notch on the second to locate the other pair of points you should connect. This drawing shows the general idea, but not what your finished drawing will look like. Notice that in this illustration,  $5 + 1 = 4 + 2 = 3 + 3$ . This helps to insure that you haven't skipped any marks. If you have 22 on a side, you would be dealing with  $22 + 1 = 21 + 2 = 20 + 3$ , etc. When you complete the drawing, you are sure to see a curve, even though you know that all your lines are straight. (Actually your lines are tangents to a hyperbola, one of the conic sections, and when the points of tangency are close together, you think you are seeing the curve itself.)

Colored cord or yarn shows up well (rk).



Young children might have the outline prepared on a piece of cardboard or tag, punch holes for the points on the side of the angle, and let children use colored cord or yarn. Older children can start with their own angle...use sharp colored pencils or fine colored thread for more complicated designs. Can work on all four angles produced by two intersecting lines, or all 3 angles of a triangle (or angles of other polygons) for fascinating designs. Experimentation should be encouraged.

III. REAL NUMBER OPERATIONS  
 --- page 12  
 GAMES, PUZZLES, FUN THINGS

ANOTHER WAY TO CURVE STITCH... from  
 April Fool Math by Dr. Jo Phillips,  
 April 1968 Instructor p. 88

Start with a circle, preferably one  
 of at least 6 inches diameter. Mark  
 it off evenly all the way around,  
 every 10 or 15 degrees. You get a  
 beautiful design if you connect each  
 mark with the five closest to it,  
 moving clockwise. See what you get!  
 (You are drawing, or stitching, tan-  
 gents to circles this time.) White  
 thread on blue paper is especially  
 effective. Alter or augment these  
 instructions, being very skillful in  
 your use of colors, and you may even  
 get a design that appears to expand  
 and contract.

FUN FOR 3rd grade and Up

Using 7 for H, everywhere H occurs,  
 and for different numbers, break  
 this code. If you do it correctly,  
 the numerical answers will check.  
 It does not matter which of these a  
 pupil does first. If he has a lot  
 of work to do in solving the other  
 two after doing the first, he has  
 been fooling himself.

a) AHAHA	b) TEHAW	c) \$AHA.HA
+TEHE	-TEHE	+TE.HE
-----	-----	-----
TEHAW	AHAHA	\$TEH.AW

(Hints, to be given only if necessary:  
 T + H = 9 Why? A + E = 6. T = A + 1  
 Why? One of the solutions - TEHAW  
 is 52746.)

ARRANGING NUMBERS.....

1 2 3 4 5 6 7 8 9 = 100

Insert proper symbols so that the  
 above 9 digits actually do = 100.

(Answer:  
 $(1+2+3+4+5) \times 6 - 7 + 8 + 9 = 100$ )

ARRANGING NUMBERS (continued)

Arrange 5 fives such that they = 100

( Answer:  $(5 \times 5) - 5 \times 5 = 100$  )

Arrange six sevens so they equal 100

(Answer:  $\frac{777 - 77}{7}$  )

Arrange seven sevens so they equal 100.

(Answer:  $7 + 7 \times 7 + 7/7 + 7/7$  )  
 ( )

A MOST UNUSUAL MAGIC SQUARE...from April  
 Fool Math by Dr. Jo Phillips, April 1968  
 Instructor, p. 88

Not only do all rows, columns, and diagonals  
 add up to the same number, 19,998, but if  
 you hold it upside down, or even look at  
 its image in a mirror, you will see a square  
 for which 19,998 is the "magic" number. Be-  
 sides, its title reads the same, right side  
 up, upside down, or in a mirror.

IXOHOXI			
8818	1111	8188	1881
8181	1888	8811	1118
1811	8118	1181	8888
1188	8881	1818	8111



REVIEW ADDITION AND SUBTRACTION WITH MONEY &  
REVIEW CONSUMER WORD PROBLEMS INVOLVING MONEY

V. Money

7-8 REMEDIAL

1. Given a list of 5 problems of indicated operations, the student will correctly perform the operations on 4 of the 5 problems.

AW 1651 - p. 76  
AW 1661 - pp. 46-47; 54; 87; 129.  
AW 1671 - pp. 86-87; 96; 116-117; 291; 303;  
329-330.

2. Given 5 word problems involving purchasing items, the student will correctly answer 4 of the 5 problems.

HRW Bernstein et al: TROUBLESHOOTING MATH.  
SKILLS c 1969, Chapt. 13 (p. 392)

Readiness: Place value concepts;  
familiarity with decimal fractions;  
money recognition.

REVIEW BASIC TERMS & CONCEPTS OF GEOMETRY

VIA. Plane Geometry

7-8 REMEDIAL (Optional)

1. Given a list of basic geometric figures and corresponding parts, the student will identify with 70% accuracy the figure or part with its corresponding name.

AW 1651 - pp. 18-19; 48-49; 80-81; 112-113;  
174-175; 206-207; 244-245.  
AW 1661 - pp. 26-27; 62-63; 96-121; 136-137;  
150-151; 174-175; 214-215; 234-235;  
266-267; 286-287.  
AW 1671 - pp. 28-29; 51-52; 68-69; 80-81;  
128-129; 130-165; 186-187; 230-231;  
272-273; 282-283; 340-341.

Readiness: Point, line concepts.

Cyclo-Teacher H 61 through H 66.

311 Transparencies for figures.

Make own blocks and squares.

Use Activities for making own models of  
other figures.

RECOGNITION OF GEOMETRIC SHAPES, INCLUDING  
LINES, AND THEIR TERMINOLOGY

VI. Plane Geometry

7 - Optional

1. Student is able to match a list of 25 geometric shapes and their parts with the proper names with 80% accuracy.

HRW 7, pp. 307-343.

Readiness: Familiarity with line concepts.

STUDENT WILL HAVE A WORKING KNOWLEDGE OF  
CENTRAL AND INSCRIBED ANGLES.

VI. Plane Geometry

8 S

1. Given diagrams of ten angles, the student will successfully match each angle with its definition or with its corresponding arc or measure, as indicated, with 80% efficiency (8 out of 10).

HRW 8, pp. 122-125

Student will learn definitions for "semi-circle," "intercept," "chord," "inscribed," and notation for arc

Student learns a new way to construct  $90^\circ$  angle by inscribing an angle within semi-circle.

This technique can be adapted to construct line segments of lengths like  $\sqrt{3}$ .

Readiness: Understanding of angle and circle concepts.

# MEASUREMENT OF GEOMETRIC SHAPES

## VI. Plane Geometry

7-Optional; 8 S

1. Student is able to compute areas, volumes, or perimeters, as indicated, from a set of 25 problems (including word problems) involving measurements of geometrical shapes, with 80% accuracy.

Readiness: Ability to recognize geometric shapes and their parts.

HRW 7, pp. 345-390

HRW 8, pp. 122-148

Harcourt, Brace & World, Osborn et al: EXPLORING ARITHMETIC 8, pp. 171-194 A. (Especially good for formulas needed.)

HRW, Nichols: PRE-ALGEBRA MATHEMATICS, pp. 306-335. (Includes good cumulative test also.)

# STUDENT WILL BE ABLE TO CONSTRUCT, USING ONLY COMPASS AND STRAIGHT EDGE, THE FOUR BASIC CONSTRUCTIONS.

## VI. Plane Geom. Constr.

8 S

1. Given a compass and straight edge, the student can reproduce the basic constructions for:
  - (1) bisecting a line segment;
  - (2) constructing an angle similar to a given angle;
  - (3) bisecting a given angle; and
  - (4) constructing the distance from a point to a line segment; all with 100% accuracy.

Readiness: Manipulative familiarity with compass; working knowledge of line-ray relationship and set concept.

HRW 8, pp. 101-148.

Student becomes familiar with terms of "perpendicular," "arc," "bisect," "angle," "diagonal," "concurrent," "polygon," and "regular polygon."

"Construction" contrasted with drawing or sketching.

## SUPPLEMENTARY ACTIVITIES

**FILM LOOP - 'FROM A TO Z'**  
 Encyclopedia Britannica  
 Super 8 MM Color No. S-80184 \$17.60

or

8 MM Color No. R-80184 \$16.00  
 Concepts of line and point symmetry  
 are illustrated using a selection  
 of symmetrical and asymmetrical  
 capital letters.

**FILM LOOP - POINT-LINE-PLANE**  
 Encyclopedia Britannica  
 Super 8 MM Color No. S-80160 \$22.00

or

8 MM Color No. R-80160 \$20.00  
 Real life situations - concrete  
 illustrations of points, straight  
 lines, and planes.

**FILMSTRIP - GEOMETRY**  
 Society for Visual Education, Inc.  
 (subsidiary of the Singer Co.)

Modern Geometry for Jr.-Sr. High - Color  
 542-1 Distance and Betweenness (55 frames)  
 542-2 Points, Lines and Planes (includes  
 convex sets and separation proper-  
 ties) (58 frames)  
 542-3 Angles - linear pair, supple-  
 mentary, complementary, per-  
 pendicular, vertical, etc.

**RENTAL FILM - DYNAMICS OF THE CIRCLE**  
 U. of Mo. 1968 New Films  
 14 min. (presumed black and white)  
 \$3.75 for 1-3 days.  
 To state and illustrate definition  
 of circle, radius, chord, diameter,  
 arc, and central angle. To illus-  
 trate dynamic effects of variation  
 of one upon the other.

**EASY-TO-PASTE SOLIDS** by M. Stoessel  
 Wal, Arith. Teacher Oct. 1965,  
 pp. 468-471  
 Easy diagrams for constructing oak-  
 tag models (solid) of Tetra-hedron,  
 Octahedron, Icosahedron, Hexahedron,  
 60-faced solid. These can be used

for mobiles... or painted and glittered  
 for Christmas decorations except 'many  
 states have new laws forbidding this.'

American Book Co., Deans et al, **STRUCTURING  
 MATHEMATICS 7**, c 1966, Teachers Edition:

Chapt. 4, pp. 91-118, Points, Lines,  
 Planes, and Space.

Chapt. 9, pp. 257-300, Polygons and  
 Right prisms.

Chapt. 11, Circles and Right Circular  
 Cylinders, pp. 323-342

These chapters are good for 'Sharp  
 Thinkers,' stimulating practice  
 problems, and good reviews and chapter  
 tests.

Allyn & Bacon, Stokes et al, **ARITHMETIC IN  
 MY WORLD 8**, pupil pp. 151-159 or pp. 99-  
 101 in Teacher Edition, for use of con-  
 gruent triangles, similar triangles, and  
 their use in solving word problems.

Laidlaw, McSwain et al, **MATHEMATICS 8**, c  
 1965, pp. 83-98.

Chapt. 8, pp. 149-166, Points and Lines  
 Includes good evaluations and review  
 material.

Chapt. 10, Constructing Congruent Line  
 Segments, p. 185..., Triangles, p. 193,  
 and Parallel lines, thru p. 204.

Chapt. 11 - Perimeter of a polygon, Areas,  
 Volumes. Good thought-provoking problems  
 in Self-Evaluation on p. 230.

# SUPPLEMENTARY ACTIVITIES

Amer. Book Co., Deans et al,  
EXTENDING MATHEMATICS 8, c 1966,  
Chapter 5, pp. 129-172  
This chapter parallels our Holt-  
Rinehart-Winston text in many  
senses and gives excellent thought-  
provoking exercises for practice;  
brings in set union idea for tri-  
angles, a good concept of measure-  
ment of a line segment, suggestions  
for research topics. Bisection of  
an angle is shown on page 169, #28.  
Contains excellent review pages  
170-171, as well as a good Chapter  
Test, p. 171-172.  
See also Same Series, Grade 7, en-  
titled Structuring Mathematics.  
Chapter 11, p. 323, on Circles and  
Right circular cylinders could also  
be useful.

World Book Co., GROWTH IN ARITHME-  
TIC 8 Revised, Teacher's Edition,  
c. 1957:  
p. 128 - classifying nature designs  
into geometrical shapes  
p. 134 - geometric shapes into  
designs  
p. 129 - To Help You Remember is  
summary of geometric  
shapes together with  
salient points of angles,  
sides, etc. properties.  
p. 130 - Recognizing geometric  
shapes in every-day articles.  
p. 131 - Symmetry in every-day  
132 articles.  
pp. 136-137 and thru p. 147, on  
triangles and angles.

## GEOBOARDS

"Notes on Geoboards" and "Geoboard  
Geometry"

CG-10 \$1.50 (believe for both pamphlets)

CG-9 Cuisenaire Geoboard \$1.50

Cuisenaire Company of America, Inc.  
12 Church Street  
New Rochelle, New York 10805

## VII. MATHEMATICAL SENTENCES

FILM LOOP - MATHEMATICAL SENTENCE  
Encyclopedia Britannica  
Super 8 MM Color No. S-80078 \$17.60

or

8 MM Color No. R-80078 \$16.00

Compares word sentences to  
number sentences.

'Is' compares to 'Equal.'

'Is Not' compares to 'Not Equal to'.

FILM LOOP - "INEQUALITIES"  
Encyclopedia Britannica  
Super 8 MM Color S-80086 \$17.60

or

8 MM Color R-80086 \$16.00

Laidlaw, MATHEMATICS 8, McSwain  
et al, c 1965, pp. 83-98.  
Chapter 5 deals with Translating  
Open Expressions and English  
Phrases into mathematical and  
using such expressions to solve  
word problems. This is excep-  
tionally good for what seems to  
prove a very difficult activity.

Chapter 4, pp. 61-72, contains  
Sentences about Numbers, Order  
Relations, Open Sentences, Re-  
placement Sets, Solution Sets,  
etc.

INTRODUCTION TO CONSTRUCTION AND INTERPRETA-  
TION OF VARIOUS KINDS OF GRAPHS AND LATTICES.

VIII. Graphing

7 Optional

1. Given 15 sets of ordered pairs, student can construct various types of graphical representation as specified, with 80% accuracy (12 out of 15).

HRW 7, pp. 391-419

Readiness: Concept of ordered pairs and set concepts.

STUDENT CAN GRAPH STATISTICAL DATA INTO  
PICTORIAL FORM.

VIII Graphing, Pictorial

8 S

1. Given 5 sets of data, some in word problem form, student can graphically represent the data in circle, bar, line, or picture graph form, for 4 out of the 5.

HRW 8, pp. 47-54.

Institute of Life Insurance booklet on "Sets, Probability, and Statistics" for examples (1964 issue).

Readiness: Student needs to be able to interpret statistics as ordered pairs.

HRW, Bernstein et al: TROUBLESHOOTING MATHEMATIC SKILLS, pp. 289-295 (includes maps as graphs).

Webster, Osborn et al: EXPLORING ARITHMETIC 8, pp. 208-213.

STUDENT WILL BE ABLE TO GRAPH SETS OF POINTS  
EITHER ON AN ARRAY OR LATTICE OR ON  
COORDINATE PLANE

VIII. Graphing

85

1. Given 5 lists of ordered pairs, student can construct graphical representations for 4 out of the 5, on lattice or coordinates as indicated.

Readiness: Student needs number theory concepts and a working knowledge of directed numbers.

HRW 8, pp. 311-374

Student is introduced to ordered pairs on graphs by means of point location on various arrays. (Army grid in existence for entire world by locating "points.")

Arrays are extended into coordinates -- student learns terms of "axis," "quadrants," "symmetry," "slope."

Student practices graphing, including graphing solutions sets for two conditions, and graphing subsets.



REVIEW NUMBER LINE GRAPHS AND REINFORCE  
PLOTING COORDINATE PAIRS

VIII. Graphs

8 REMEDIAL(Optional)

1. Given a number line, the student can locate a given point on the line.
2. Given a set of ordered pairs, the student will plot the set of ordered pairs on a coordinate system with 70% accuracy (7 out of 10).

AW 1671 - pp. 355-365.

Use columns and rows for locating coordinate graphing points.

HRW, Bernstein et al, TROUBLESHOOTING MATH.  
SKILLS c 1969, pp. 355-365.

## VIII. GRAPHS

### SUPPLEMENTARY ACTIVITIES

Heath Co., Fehr Schult, MATHEMATICS  
IN LIFE 2, p. 421.

General rules on graphing.  
Also good on other 'measures.'

GRAPH THESE POINTS (to make a six-pointed star)  
p. 175, ELEMENTARY MATH., 2nd Ed., Harcourt,  
Brace & World, c. 1968, 1966.

A. (8,2)

B. (6,5)

C. (2,5)

D. (4,8)

E. (2,11)

F. (6,11)

G. (8, 14)

H. (10, 11)

I. (14, 11)

J. (12, 8)

K. (14, 5)

L. (10, 5)

To make a steamboat, graph:

A. (4,3)

B. (2,5)

C. (5,5)

D. (5,6)

E. (7,6)

F. (7,8)

G. (8,8)

H. (8,6)

I. (9,6)

J. (9,8)

K. (10,8)

L. (10,6)

M. (13,6)

N. (13,5)

O. (16, 5)

P. (13, 3)

COORDINATES TO TRACE A  
CHRISTMAS TREE

Math. Tchr. Dec. 1968, p. 764.

MAKING MODELS ....

of 3-dimensional geometric shapes (triangular prism, cube, balls of clay).

Arith. Tchr. Oct. 1965, 'Geometric concepts in Grades 4-6' by Dora Helen Skypek, pp. 443-449.

STUDENT IS INTRODUCED TO BASIC CONCEPTS AND  
BASIC TERMINOLOGY OF PROBABILITIES

IX. Probability

8 S

1. Given a specific experiment, the student can write the sample space for the experiment and can also identify the probability for a given specific outcome.

Readiness: Familiarity with grouping principles.

HRW 8, pp. 29-39

(See also Chapter 15 in Addison-Wesley new Book VI, pp. 312-321.)

Transparencies for overhead on both concepts and terminology.

Petroleum company pamphlet on probabilities.

Red booklet by Life Insurance Institute (1964), SETS, PROBABILITIES, AND STATISTICS.

Laidlaw, JR. HIGH MATHEMATICS 8, pp. 313-331 (includes independent tests, etc.) (1968) Chapter 12.

## IX. PROBABILITY

American Book Co., EXTENDING  
MATHEMATICS 8, by Deans et al,  
c 1966, Chapter 12.

This entire chapter is very  
good on all basic aspects of  
probability including practice  
exercises, suggested research  
topics, and excellent review  
and chapter test on pp. 415-416 as  
well as an interesting cumulative  
test on pp. 417-419.

HEXSTAT, Probability demonstrator  
Harcourt, Brace, World, Inc.

This little gadget utilizes  
a honeycomb of passages for  
tiny steel beads and one copper  
bead, from top to bottom.  
Fascinating!

STUDENT GAINS EXPERIENCE IN WORKING WITH RATIOS  
OF RIGHT TRIANGLES, WHICH LEADS TO DEVELOPMENT  
OF TRIGONOMETRIC FUNCTIONS OF SINE, COSINE, TAN-  
GENT AND COTANGENT

X. Trigonometry  
(Rt. Triangle)  
8 S

1. Given two similar triangles, the student can determine the missing parts either by proportion or by trigonometric function. Given situations may be in the form of word problems.

Readiness: Needs understanding of right angle, right triangle, and proportion.

HRW 8, pp. 149-192.

Student learns what two conditions are necessary for similar right triangles.

Student learns to identify complementary angles.

Student learns how to use trigonometric tables and how to interpolate for in-between values.

Student learns definition of trigonometric functions in terms of ratios.  
(No secant or cosecant is included.)

Student learns how to apply these techniques in problem-solving and practical applications.

X. TRIGONOMETRY - RIGHT  
TRIANGLE

World Book Co., GROWTH IN  
ARITHMETIC 8, Revised, Teacher's  
Edition, c 1957.

PP. 248-251, Ratios of similar  
triangles, using similar triangles,  
and solving problems by similar  
triangles.

STUDENT WILL BE ABLE TO DEVELOP SUM, NUMBER OF  
TERMS, OR SPECIFIC TERM OF AN ARITHMETIC  
PROGRESSION

## XII. Progressions

8 S

1. Given 3 arithmetic progressions, student will compute the correct sums for 2 out of the 3.
2. Given 3 arithmetic progressions, student will compute the correct number of terms for 2 out of the 3.
3. Given 3 arithmetic progressions, student will compute the specified term correctly for 2 out of the 3.

**Readiness:** Student must be able to utilize formulas and must be familiar with set concepts.

HRW 8, pp. 85-98.

Student develops formula for sum of series:

$$S = \frac{a_1 + a_n}{2}$$

Student develops formula for last term or specific term (found in Chapt. Review):

$$a_n = a_1 + (n - 1) d ;$$

From the above formula, notation for the number of terms is developed as

$$n = \frac{a_n - a_1}{d} + 1,$$

where S = sum of series  
n = number of terms,  
d = difference between any two consecutive terms,  
 $a_n$  = last or nth term of series,  
and  
 $a_1$  = first term of a series.

RENTAL FILM - MEAN, MEDIAN, MODE

U. of Mo. 1968 New Films

13 min. Color \$5.00 for 1-3 days.

Explains that the meaning of average includes the concepts of median, mode, and arithmetic mean. Uses meaningful situations to develop use and computation of average.



I SETS

G 9 R

1. The student can show familiarity and know correct mathematical meaning of the language and symbolism of sets to include the following:

Roster, rule, element, member, null or empty set, subset, one to one correspondence, equivalent sets, finite sets, infinite set, replacement set, variable, domain.

1. A pretest can be used to determine if further individual explanation and practice is needed. Suggest a list of exercises be selected and study the reference material given below.

2. "Basic Text", pages 18-34

PROPERTIES (TO IDENTIFY AND UNDERSTAND)

II Number Theory

G 9 S and R

1. The student will show a recognition of the properties of real numbers as listed below:

Commutative (Add. & Mul)  
Associative (Add & Mul)  
Distributive  
Closure  
Special properties of zero and one, with respect to addition and multiplication.  
Properties of equality  
Reflexive  
Symmetric  
Transitive

1. Use specific examples to identify student recognition.

$1 + 2 = 2 + 1$  or  $\Delta + \square = \square + \Delta$   
as an illustration of the commutative property with respect to addition.

2. The student, by means of an inventory test, will indicate any properties which need to be reviewed, studied or introduced for the first time to a few students whose background may not have included all properties

3. "Basic Text" pages 47-51  
" 55-56  
" 61, 67, 68, 71, 73  
83, 84

pages 87-88 gives an excellent summary list of statements of axioms, properties and theorems.



**PREPARATION FOR ALGEBRA  
BASIC DEFINITIONS AND CONCEPTS**

**III REAL NUMBER OPERATIONS**

G 9 R

1. The student can demonstrate his recognition and understanding of the following mathematical terms, signs, and symbols:

Whole number, real numbers to include rational numbers, use of the number line, the following signs:

$=, <, >, \div, +, -, <, >, \leq, \geq,$   
signs of inclusion,  $( ), [ ]$ ,  
powers, base, exponent and order of operation.

1. Use a pretest to determine if any additional time is necessary to further reinforce any apparent weakness.
2. "Basic Text": pages 1-16  
(Hereafter, "Basic Text" will refer to: Modern School Mathematics, Algebra I; Dolciani, Wootton Beckenbach, Jurgensen, Donnelly; Houghton Mifflin Co., 1967.)
3. Use Filmstrip #206 for individual instruction. (Unless otherwise indicated filmstrips are those located in the Resource Center, SHS)

**ADDITION OF POLYNOMIALS**

Includes Subtraction as the Additive Inverse

G 9 R

**III Real Number Operations**

2. The Student will know that he is able to add algebraic polynomials and is familiar with and understands the mathematical vocabulary associated with the addition properties as listed below:

Closure, addends, coefficient, like terms, binomial, trinomial, simple form, degree, number line, identity element of addition and the additive inverse.

1. "Basic Text" Pages 47- 64, R  
77- 82, R  
95-106, S
2. FS (Film Strip) # 001, "Meaning of signed numbers and How to Add Them", may be used for individual student study or Classroom use:

## PRODUCTS OF POLYNOMIALS

## III REAL NUMBER OPERATIONS

G-9 S

2. The student will show he can multiply a monomial and a polynomial and will apply the distributive

1. Sufficient practice should be given in developing the ability of the student to find the product of two binomials at sight.

2. "Basic Text"; pp. 267-279

### 2. Application

The student can show mastery of finding products of binomials by solving problems requiring multiplication of binomials to solve the equation.

1. "Basic Text" pp. 276-277

## FACTORING OF POLYNOMIALS APPLICATION

## III REAL NUMBER OPERATIONS

G 9 S

1. The student will be able to factor polynomials by applying previous knowledge of products.

1. The general development of factoring should be:
  - a. Find greatest monomial factor.
  - b. Recognizing a pattern if it exists and factoring accordingly.

The patterns are:

$$a^2 - b^2 = (a + b)(a - b)$$

$$a^2 + 2ab + b^2 = (a + b)^2$$

$$a^2 - 2ab + b^2 = (a - b)^2$$

2. The student will recognize and apply the 3 general factor patterns which occur frequently.

2. "Basic Text", pp280-300

3. The student will apply factoring to polynomials formed to solve various types of word problems.

**MULTIPLICATION OF POLYNOMIALS**  
Includes Division as the Multiplicative  
Inverse

**III REAL NUMBER OPERATIONS**

G 9 S

1. The student will be able to multiply and divide polynomials of 1, 2, and 3 terms with degree 0 to 3 and with coefficients of real numbers.

Readiness: Previous ability should consist of multiplication and division of rational real numbers.

1. The axiom of the multiplicative inverse and the relationship between multiplication and division shall be established.

Theorem: For all real numbers  $a$  and all nonzero real numbers  $b$ ,  $a \div b = a(\frac{1}{b})$ , which gives the definition of division and implies that the set of real numbers is closed under division.

2. Reference and exercises-"Basic Text" pp. 106-115.

3. For reinforcement the individual may be referred to Filmstrip #002, "Multiplication of Signed Numbers", and #004, "Absolute Value". Filmstrips are located in the Resource Center.

## DIVISION OF POLYNOMIALS

### III REAL NUMBER OPERATIONS

G 9 S

1. The student will review the laws of exponents with respect to division and learn the meaning of the zero and negative exponents.

a.  $\frac{a^m}{a^n} = a^{m-n} \quad m > n$

b.  $\frac{a^m}{a^n} = \frac{1}{a^{n-m}} \quad m < n$

c.  $\frac{a^m}{a^n} = 1 \quad \text{if } m = n$

d.  $a^0 = 1$

e.  $a^{-n} = \frac{1}{a^n}$

2. The student will be able to reduce (simplify), divide and combine (add) rational polynomial expressions.

1. Practive application of the exponent laws with monomial expressions.

2. "Basic Text ", pp 307-312, 324-325.

1. The application of the principles of fractions will be reviewed and applied to algebraic rational expressions.

2. "Basic Text", pp. 328-338, 313-323.

## SUMS OF REAL NUMBERS WORD PROBLEM SOLVING

### III REAL NUMBER OPERATIONS

1. The student will translate a verbal statement into a mathematical statement, indicating sums of real numbers and simplify the expression to a single real number correctly.

1. "Basic Text", pp. 65-66

2. Review of Filmstrip #001 "Meaning of Signed Numbers and how to add them.

## RATIONAL NUMBERS IN PROBLEMS

### III REAL NUMBERS OPERATIONS

G 9 S

1. The student will write equations containing fractions from facts of problems and solve for the solution sets.

Readiness: The student should be familiar with ratio and fractional expressions, factoring and equation solving.

1. Special classes of problems possibly introduced at this time involving fractions are: mixture, profit and loss, cost, interest, work and time.
2. The multiplicative property of equality will be emphasized as the means of transforming open sentences with fractional coefficients into equivalent sentences without fractions.
3. The importance of the check must be emphasized in terms of the original equation.
4. "Basic Text", pp. 345-365

## PROBLEMS INCLUDING INEQUALITIES

### III REAL NUMBER OPERATIONS

G 9 S-R

1. The student shall be able to correctly translate into algebraic language, word problems containing expressions of greater than, less than, at least, more than, as well as the equality statement.

Readiness: The student is familiar with the uniform motion relationship, cost relationship and the meaning of complementary angles.

1. After forming the mathematical sentence, the student will continue use this knowledge of solving the equation to find the solution set and make a check of the numerical answer.
2. "Basic Text", pp. 169-180.
3. Filmstrip #9427, "Solving Equations" St. Nley Bowmar Co.
4. Filmstrip #210, "Solving Equations" 2nd part, Resource Center, ~~SHA~~
5. "Programmed Units in Mathematics" by Denmark and Sample, Houghton Mifflin Co., will make an excellent source of supplemental help for 1st year Algebra students and good reinforcement for Alg II students. They may be obtained in Work Problems, Mixture Problems, Motion Problems and percentage at small cost.  
(See 1978-69 catalog)

WORD PROBLEM SOLVING  
Writing The Equation

III REAL NUMBER OPERATIONS

1. The student shall write an algebraic expression or equation from a given problem situation.

1. Example: A total amount of 282 gallons of oil is contained in 5 drums, each containing  $x$  gallons and a 6th drum containing 32 gallons.

$$5x + 32 = 282$$

- gal. gal. total gals.  
2. Equation should be a correct statement of fact.  
3. "Basic Text", pp. 120-124.

PROBLEM SOLVING  
SOLVING THE EQUATION

III REAL NUMBER OPERATIONS

1. The student shall find the root or solution to an algebraic equation with one variable to the first degree.

Readiness: The student can write the proper equation that expresses correctly the problem situation.

1. Equations of this type shall be solved by applying the transformations of substitution, adding multiplying, subtracting and dividing to produce an equivalent equation whose solution set can be found by inspection.  
2. The importance of checking the apparent solution should be emphasized.  
3. The sequence of problem solving should be reemphasized.  
a. Read and decide what is to be found.  
b. Choose a variable, identify it.  
c. Write an open sentence using relationships found in the problem or indicated by the nature of the problem.  
d. Solve the open sentence.  
e. Check with requirements of problem.  
4. "Basic Text", pp. 116-136.  
5. Filmstrip #013, "Problem Solving" will provide supplemental instruction.

OPERATIONS WITH RADICALS  
Irrational Numbers

III REAL NUMBER OPERATIONS

G 9 S

1. The student will apply the properties of roots to radicals in order to simplify them and perform the four fundamental operations with sets of radicals.

1. The properties of radicals;  

$$\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}; \frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$$
 help to simplify radicals to better facilitate their combination
2. "Basic Text", pp. 437-448.
3. To solve a radical equation
  - a. Isolate the radical in one member
  - b. If square root, square both members.
  - c. "Basic Text", pp. 451-461.

IRRATIONAL NUMBERS

III REAL NUMBER OPERATIONS

1. The student will extend his concept of irrational numbers and be able to approximate their decimal values.

1. Emphasis should be placed on squares, square roots, the decimal notation for rational and irrational numbers, and to recognize and produce them
2. "Basic Text" pp. 419-430.

SYSTEMS OF LINEAR EQUATIONS  
PROBLEM SOLVING

III REAL NUMBER OPERATIONS

G 9 S

1. The student will translate word problems into equations using two variables and correctly solve the system.

1. The translation of set of facts into a pair of equations in two variables should be first practiced.  
Example: The sum of 2 numbers is 40  
and their difference is 14.

$$\begin{aligned}x + y &= 40 \\x - y &= 14\end{aligned}$$

2. Various formulas to be used should be reviewed.

Uniform motion  $r \cdot t = C$

Cost formula  $n \cdot P = C$

Perimeter  $P = \text{Sum of lengths}$

Relationship of parallel  $F_1 + F_2 = R$   
and opposite forces  $F_1 - F_2 = R$

3. Emphasize many problems can be easily solved by using 2 variables
4. "Basic Text", pp244-251.
5. The Mathematics Teacher.  
Jan. 1960, p. 12.



ALGEBRAIC EXPRESSIONS AND SENTENCES  
WITH VARIABLES

VII MATHEMATICAL SENTENCES

G 9 R

1. The prospective algebra student will show his ability to transform word phrases into mathematical expressions using variables, quantifiers and open sentences.

Readiness: The student should be able to solve open sentences in one variable.  
(equations and inequalities)

1. If additional study and practice is needed, study and use selected exercises, "Basic Text", pp. 35-45.

INEQUALITIES IN MATHEMATICAL  
SENTENCES

VII MATHEMATICAL SENTENCES

G 9 S

1. The student will be able to apply the axioms, theorems, and transformations of inequality to produce a solution set.

Readiness: The student should be familiar with the symbols of inequality, and with the axiom of comparison or Trichotomy.

1. Transformations used to affect the solution set are substitution, addition, subtraction, multiplication and division.

2. Particular attention should be paid to the reversing of order of the inequality when multiply or dividing each member by the same negative number.

3. "Basic Text", pp. 145-151, 157-165.

4. Filmstrip #9426, "Inequalities", Stanley Bowmar Co.

## INTERPRETATION OF DATA AND PROBABILITY

### IX. Statistics

G-9-C

- |  |  |
|--|--|
| <ol style="list-style-type: none"><li>1. The student will become familiar with the collection, organization and interpretation of data, and probability.</li></ol> | <ol style="list-style-type: none"><li>1. Specific topics to be accomplished will be to:<ol style="list-style-type: none"><li>(a) Arrange data in forms that indicate frequency, cumulative frequency, percentile, mode and mean.</li><li>(b) Find variance and mean deviation.</li><li>(c) Find elementary probability of an event, <math>P(E)</math>.</li></ol></li><li>2. Basic text, pp. 587-616.</li><li>3. Filmstrip #209, "An Introduction to Probability, Resource Center, SHS.</li><li>4. "Non-intuitive Probability, "Math Teacher, May 1969, p. 361.</li><li>5. "Gambling Doesn't Pay."--Probability Math Teacher, March 1967, p. 200</li><li>6. Use simple experiments such as coin tossing, card drawing, objects in a container, etc.</li></ol> |
|--|--|

## ANGLES, FUNCTIONS, APPLICATION, VECTORS

### X. Trigonometry

G9-D

- |   |   |
|---|---|
| <ol style="list-style-type: none"><li>1. The student will learn how angles may be used in many practical problems, the sine, cosine and tangent function, use of tables, and the use of vectors to represent displacement, velocities and forces.</li></ol> | <ol style="list-style-type: none"><li>1. Since the student has some previous experience with triangles, the time may vary and some effort should be made to reinforce and add to his previous knowledge of trigonometry.</li><li>2. Basic text, PP. 545-581.</li><li>3. "How Shall We Define Angle"? The Math Teacher, Jan. 1967, p 18.</li></ol> |
|---|---|

## PERMUTATIONS

IX. Statistics

G11-S

1. The student can compute the number of arrangements or permutations of a set of objects under varying restrictions.

1. Develop the basic relationship or formula for computing permutations:  $P_n = n!$  where  $n$  = number of objects and  $n! = n(n-1)(n-2) \dots (2)(1)$ . Use three objects A, B, C and show how they can be arranged in a row.  
(ABC), (ACB), (BAC), (BCA), (CAB), (CBA)  
 ${}_3P_3 = 3! = 3 \times 2 \times 1 = 6$  arrangements
2. Place the restriction that a lesser number of objects will be used than those available.  
 ${}_nP_r = n(n-1)(n-2) \dots (n-(r+1))$
3. Involve the fundamental principle of counting by examples such as how many different odd 3 digit numbers can be formed from the digits 1-9, if no digits are repeated. Using place systems.  
Fill the units place first. This gives 5 choices, leaving 8 for the tens place and 7 for the hundredths place, giving  $(8 \times 7 \times 5)$
4. References: Algebra and Trigonometry  
Book 2; Dolciani, et.al.

## SPACE GEOMETRY - 3 VARIABLE

### XI. Geometry

1. The student will learn about linear equations in 3 variables and how they may be graphed as planes in space.

The solution to a set of 3 equations in 3 variables should be found correctly and some verbal problems should be solved.

1. This should include ordered triples and their graphs, space coordinate, graph of the plane ( $ax + by + cz = d$ ) in space, use of traces, and lines in space.

2. Basic text, pp. 510-544.

## THE QUADRATIC FORMULA

### XIII. Functions

G9-S

1. The student will be able to solve a quadratic equation by completing the square and be shown how the quadratic formula is obtained.

2. The student will learn the quadratic formula by using it to solve equations.

1. Emphasis will be placed on the fact that completing the square enables one to find irrational roots when factoring will not work. (rational roots).

2. Basic text, pp 448-452

1. The discriminant will be emphasized ( $b^2 - 4ac$ ) and how it indicated characteristics of the roots.
2. Application should be made to problems where it is necessary to use the formula to simplify finding the solution.
3. Basic text, pp. 448-457

# SLOPE, INTERCEPT, AND EQUATION OF THE LINE

## XI ANALYTICAL GEOMETRY

G 9 S

1. The student can find the slope of a line and use the slope and intercept concept to determine the equation of a line.

Readiness: Be able to obtain the coordinates (ordered pairs) of 2 points on a line graph on the coordinate plane. The terms abscissa and ordinate as names of the two parts of the ordered pairs may be introduced.

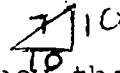
1. Concept of slope should be introduced as a ratio of vertical change to horizontal change. If the coordinate axes are named  $x$  and  $y$  and two points  $A$  and  $B$  have coordinates  $(x_1, y_1)$ ,  $(x_2, y_2)$  respectively, then the slope ( $m$ )

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Slope should be compared to the pitch of a roof in construction and percent grade of a road surface.

Emphasis (A 100% grade is not vertical but equal to a slope of 1.

$$1 = 100\%$$



$$M = \frac{1}{10} = 10\% \text{ grade}$$

2. Show that the intercept will mean the  $y$  intercept designated by  $(b)$ .
3. A line can be shown to have a slope and an intercept. Use graphs.
4. A line can be represented by the equation  $y = mx + b$ , called the slope-intercept form of the straight line.
5. The equation of a line can be written if the slope ( $m$ ) and  $y$  intercept ( $b$ ) are known.
6. The  $x$  and  $y$  values of the coordinate of a point on a line will determine the  $y$  intercept when the slope is known, and the equation of the line can be determined. See 5.
7. Zero slope of the horizontal line and no slope of the vertical line should be emphasized as well as the negative and positive slope.
8. "Basic Text", pp. 206-216.
9. Filmstrip #208, "The Slope of a Line", Resource Center, SHS.

# GEOMETRY DISTANCE FORMULA

## XI ANALYTICAL GEOMETRY

G 9  
S

1. The student will recall the Pythagorean theorem, how to find the distance between two points on the number line and apply them to form the distance formula and use it to find distance between given points on the coordinate plane.

1. Practice with the theorem and its converse. Use general notation for points  $(x_1, y_1)$ ,  $(x_2, y_2)$ , etc and develop the distance formula.

2. "Basic Text" pp 431-437.

# ORDERED PAIRS EQUATIONS IN TWO VARIABLES

## XI ANALYTICAL GEOMETRY

G 9  
S

1. Student shall be able to relate the equation in two variables (first degree) to the straight line and the solution sets to ordered pairs (coordinates)

Readiness: Previous experience with the one-to-one correspondence between the set of all points of a plane and the set of all ordered pairs of real numbers.

1. The plotting of points (ordered pairs) on the coordinate plane and the equation also should be practiced.
2. The general form of the linear equation should be recognized as  $Ax + By = C$ .
3. Linear inequalities should be related to the graph and the open and closed half plane.
4. Filmstrip #9447, "Coordinate Geometry" and #9428, "Graphing Equalities" Stanley Bowmar Co.
5. "Basic Text", pp. 189-205.

SYSTEMS OF LINEAR EQUATIONS  
GRAPHIC INTERPRETATION AND SOLUTION

XI ANALYTICAL GEOMETRY

G 9  
S

1. The student can determine or estimate the coordinates of the points of intersections of 2 or more lines on the coordinate plane, and the equivalent system can be obtained, which can be used to give the solution set.

Readiness: The student should be able to easily and quickly sketch the correct graph of a linear equation

1. Given linear systems, graph and find the solution sets.

The meaning of consistent and inconsistent, as applied to a linear system should be understood.

2. "Basic Text" pp. 229-235.

3. Filmstrip # 2428, "Graphing Equations" by Stanley Bowmar Co. can be reviewed.

SYSTEMS OF LINEAR EQUATIONS  
ALGEBRAIC SOLUTIONS

XI ANALYTICAL GEOMETRY

G 9  
S

1. The student can solve systems of linear equations having 2 variables, eliminating one of the variables by substitution, addition, or subtraction and check the solution set.

Readiness: The student can multiply both members of an equation by the number necessary to make the coefficients of the same variable alike.

1. Indicate the characteristics that tell which method will most readily and easily obtain the solution set, such as, like and unlike signs, value of coefficients and when substitution is indicated by one term having the numerical coefficient of (1) one.

2. "Basic Text", pp. 235-244.

## INEQUALITIES IN TWO VARIABLES

## XI ANALYTICAL GEOMETRY

G 9  
S

1. The student can correctly graph a system of inequalities and show the set of points (intersection of half-planes), whose coordinates satisfy the system.

Readiness: The student has previously graphed the linear equation and the linear inequality.

1. Emphasize the bringing together of algebra and geometry in showing the solution.

2. "Basic Text", pp. 252-253

## LINEAR PROGRAMMING

## XI ANALYTICAL GEOMETRY

G 9  
0--8

1. The student will become familiar with the present day importance of linear programming and can find and graph the restraints, find the corner points, and indicate the polygonal convex region.
2. The student should be able to correctly interpret the graph.

1. This is the most important recent development in math of a practical application of a large part of the previous algebra. It is widely used in areas of business and industry. It shows a particular use of inequalities.
2. See "Linear Programming Problems for 1st year Algebra", The Mathematics Teacher, March, 1960.
3. "Basic Text", pp. 253-258
4. Transparency Set, #519 ACSP, by Creative Visuals would be useful.



## INDIRECT PROOF

## XII REASONING

G 9 S

1. The student shall understand the steps, reasoning and logic of the indirect proof. and properly use them in an example of an indirect proof of a hypothesis.

Readiness: The student has previously used the direct proof.

1. To write an indirect proof:
  - a. Start with the hypothesis and two possibilities-one true and one false.
  - b. Now show that one possibility (the false one) leads to a contradiction of an accepted fact.
  - c. Thus, the second(true) possibility must be the case. (The conclusion is true).
2. This indirect proof is quite often used for the proof of theorems relating to inequalities.
3. "Basic Text", pp. 151-156.
4. Filmstrip #9441, "Nature of Proof", Stanley Bowmar Co. This can be used to re-emphasize the direct proof.
5. Filmstrip #210, "Proof in Algebra" in Resource Center, SHS, 1st part.

PROOF OF ALGEBRAIC THEOREMS

XII REASONING OR LOGIC

G 9 S

1. The student will be able to begin with a hypothesis (theorem) and proceed by means of a sequence of true statements (axioms and properties) to the conclusion in a logical manner.

1. "Basic Text", pp.71-76.
2. Review of previously studied properties of real numbers such as:  
Closure  
Substitution principle of equality  
Additive Inverse  
Associative axiom of Addition  
Additive Axiom of Zero  
Transitive property of equality  
Commutative property of Addition  
Commutative property of multiplication  
Other axioms, properties and proven theorems.

COMPUTERS AND PROGRAMMING

XII REASONING OR LOGIC

G-9 O

1. The student will become acquainted with and learn to write simple programs in new programming language and study some mathematical concepts involved.

1. Symbols to indicate procedure: flow charts to describe how procedure or algorithm is to be carried out; assignments, statements and programming language are to be covered sufficiently to give students some insight into the world of the computer.
2. "BASIC TEXT", pp. 472-509
3. "History of Computers", The Mathematics Teacher, Jan. 1968, p.46.
4. Use sample punch cards, printout sheets, etc.

APPLICATION OF DIRECT AND INVERSE VARIATION

XIII. Functions

G9-0

1. The student can translate a problem into an equation and find the solution when the types of variation are combined. Proportion and/or the equation involving the constant of variation can be used.
1. Joint variation: Let  $x$  vary jointly as  $y$  and  $z$ . Gives--  
 $x/yz = k$  or  $x_1/x_2 = y_1z_1/y_2z_2$
2. Combined variation: Let  $v$  vary directly as  $T$  and inversely as  $P$ . This gives--  
 $VP/T = K$  or  $V_1/V_2 = T_1P_2/T_2P_1$
3. Emphasize the large number of applications of variation that can be found in the physical world.
  - (1) rate, time relationship
  - (2) volume of a gas
  - (3) mechanical forces, the lever, pulley, gears, meshed (transmission)
  - (4) electricity
  - (5) gravity
  - (6) Orbiting satellites and moon trips
4. Basic text, pp. 408-413.

## THE QUADRATIC AS A FUNCTION

### XIII. Functions

G9-S

1. The student will identify the quadratic  $ax^2 + bx + c$  as a function in the form  $f(x) = ax^2 + bx + c$  and further identify the graph as a parabola.

1. Quadratic functions should be graphed over the domain  $\mathbb{Q}$  and note particularly the zeros of the function. Compare to the solution of the equation  $ax^2 + bx + c = 0$ .

2. Basic text, pp. 394-400.

3. Film strip #207, "Quadratic Equation and Solutions", Resource Center, SHS.

4. "Useful Generalizations of the Concept of Functions," Math Teacher, Oct. 1959, p 444

2. The student will identify the minimum and maximum point and correlate the point with minimum and maximum value of the function. The point will also be identified as the vertex.

1. Basic text, p. 395

3. The student will understand the meaning of symmetry of the parabola and how the value of the domain at the axis of symmetry can be used to determine the vertex and the minimum or maximum value of the function.

1. Basic text, pp. 395-400.

# EXPONENTS - MULTIPLICATION

## XIII. Functions

69-R-S

1. The student shall be able to apply the laws of exponents for multiplication to indicated products of expressions containing powers. The laws are: For all real numbers  $a$  and  $b$  and all positive integers  $m$  and  $n$ :
  - (1)  $a^m \cdot a^n = a^{m+n}$
  - (2)  $(a^m)^n = a^{mn}$
  - (3)  $(ab)^m = a^m b^m$
  - (4) If  $ab = 0$ , then  $a = 0$  or  $b = 0$
1. Show that exponents can be literal numbers as well as real numbers. Use sufficient variety of examples to determine near 100% accuracy in application of the laws of exponents. Be sure that the difference between  $ab^2$  and  $(ab)^2$  is understood.
2. Filmstrip # 9423, "Exponents", Stanley Bowman Co.
3. Basic text, pp. 267-272.

## INTRODUCTION TO FUNCTION

### XIII. Functions

G9-S

1. The student will learn to recognize, write, evaluate the mathematical connotation of the term function.
1. The formal introduction to function can be based on using the following concepts.
  - (a) mapping diagram showing association ordered pairs, leading to the definition of function: The first components of different ordered pairs in a set must be different.
  - (b) If not a function, the set is a relation.
  - (c) Range and Domain are to be introduced.
  - (d) Symbols  $J$  and  $R$ , meaning integers, and real numbers respectively are to be used.
  - (e) Function notation should be emphasized.
    - (1)  $(x, 5x): x \in R$
    - (2)  $x \rightarrow 5x$
    - (3)  $f: x \rightarrow 5x$
  - (f) Be able to find value of the function at a specific value of the domain  $f(3) = 5 \cdot 3$  using  $c$ .
2. Basic text, pp. 377-387.
3. Lively Functions for Algebra One, The Math Teacher, May 1969, p 365.

## LINEAR FUNCTION - DIRECT VARIATION

XIII. Function

G9-S

1. The student is to learn to identify the function concept with the linear equation and particularly the form of the function commonly referred to as direct variation.

1. The linear equation in the form  $y=mx+b$  changed to function notation is:

$$f: x \rightarrow mx+b$$

or set of ordered pairs

$$\{(x, f(x)): f(x) = mx+b\}$$

note  $y=f(x)$  ( $m$  is slope,  $b$  is  $y$  int.)

2. Special forms would be finding zero of function (letting  $y=0$  or finding the value of the function at zero).
3. If  $m=0$ , you have the constant function.
4. If  $b=0$ , and  $m$  is nonzero, you have the function commonly called direct variation  $y = mx$ .
5. Basic text, pp. 387-390.

## PROBLEM SOLVING - DIRECT VARIATION

XIII. Functions

G9-S

1. The student can translate problems involving the linear function into equations, using ratio, proportion and correctly solve the problem.

1. Direct variation and the proportion of direct variations. One to be brought into use as many practical problems can be easily solved by the linear function, ratio, and proportion.

2. Basic text, pp. 391-394.

## THE POLYNOMIAL FUNCTION

XIII. Functions

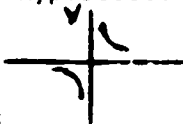
G9-

- |  |  |
|--|--|
| <ol style="list-style-type: none"> <li>The student can make a rough (graph) sketch of a polynomial function of the 3rd and 4th degree and estimate the zeros of the function.</li> </ol> | <ol style="list-style-type: none"> <li>Review the graphing of the linear and quadratic function by preparing a table of arguments and values.</li> <li>Emphasize the graph of a polynomial function with domain <math>\mathbb{R}</math> as a smooth curve. At present the student should be able to plot sufficient points to make a rough sketch.</li> <li>Basic text, pp 400-403.</li> </ol> |
|--|--|

## THE INVERSE FUNCTION

XIII. Functions

G9-0

- | <ol style="list-style-type: none"> <li>The student can identify, understand and use the function representing the inverse variation <math>xy = m</math> where the domain is understood to exclude zero.</li> </ol> | <ol style="list-style-type: none"> <li>Use <math>rt = d</math> where <math>d</math> is constant and show by a table of values that <math>r</math> and <math>t</math> vary inversely. <div style="margin-top: 10px;"> <table style="display: inline-table; vertical-align: middle;"> <thead> <tr> <th><math>r</math></th> <th><math>t</math></th> </tr> </thead> <tbody> <tr><td>50</td><td>1</td></tr> <tr><td>25</td><td>2</td></tr> <tr><td>20</td><td><math>2\frac{1}{2}</math></td></tr> <tr><td>10</td><td>5</td></tr> <tr><td>5</td><td>10</td></tr> </tbody> </table> <div style="display: inline-block; vertical-align: middle; margin-left: 20px;"> <p>Given <math>d = 50</math> miles,<br/><math>r</math> in mph and <math>t</math> in hours.</p> </div> </div> </li> <li>Graph an example of the inverse: <math>xy = 12</math> obtaining a form of the hyperbola. <div style="margin-top: 10px;">  </div> </li> <li>Basic text, pp. 403-408</li> <li>Filmstrip #203, "Inverse Variation" in resource center, SHS</li> </ol> | $r$ | $t$ | 50 | 1 | 25 | 2 | 20 | $2\frac{1}{2}$ | 10 | 5 | 5 | 10 |
|--|---|-----|-----|----|---|----|---|----|----------------|----|---|---|----|
| $r$  | $t$   |     |     |    |   |    |   |    |                |    |   |   |    |
| 50   | 1   |     |     |    |   |    |   |    |                |    |   |   |    |
| 25   | 2   |     |     |    |   |    |   |    |                |    |   |   |    |
| 20   | $2\frac{1}{2}$  |     |     |    |   |    |   |    |                |    |   |   |    |
| 10   | 5   |     |     |    |   |    |   |    |                |    |   |   |    |
| 5  | 10  |     |     |    |   |    |   |    |                |    |   |   |    |



## AN INDIVIDUAL LESSON PROGRAM FOR ALGEBRA II

This program is designed for use by an accelerated class on an individual basis, where each student can proceed at his ability rate, or it may be used for a regular class working in unison on a lesson per unit of time.

It is based specifically on the textbook presently available. This text is: Dolciani, Wooton, Beckenbach, and Sharron, Modern School Mathematics, Algebra and Trigonometry, Book 2, (Boston: Houghton Mifflin Co., 1968). Notation will follow the chapter and section numbering system of the text. For Example: 6-2 refers to Chapter 6, Section 2.

Lessons will be numbered, a brief, very specific objective will be given and detailed, suggested activities will be listed by exercise number. Odd numbered exercises will be given and the students will have available to them the answers. The following code will be used: 14/1,3,4 means page 14, Ex. # 1,3,4.  
R - Read and Study      O - Will refer to Oral Exercises  
S - Will refer to exercises of previous sections for reinforcement and review. This is the Spiral concept.

Certain procedural instructions listed below are recommended but may be changed in any manner desired to best achieve the objective.

The exercises listed in the activities may be kept in a spiral bound notebook in numerical sequence. The work will be complete, showing all significant steps, so it can be easily interpreted. Each student will check his own answers for each lesson. Exercises found incorrect in any lesson will be corrected directly following the lesson.

Periodic evaluations, by number, are indicated at the end of certain groups of lessons. A certain minimum score should be established, and any test score below this minimum should require additional work, help and instruction. The student should be responsible for requesting and obtaining this help. A retake of the particular chapter should be taken before proceeding to the next objective.

A particularly low score may indicate more extensive work and this may be accomplished by the use of "Programmed Practice". This Programmed Practice is a companion publication to the basic textbook and may be obtained from the publisher. Additional and supplemental material, visual aids, etc., should be used whenever available. Filmstrips numbered with three digits are those available at local Resource Center.

Minimum standards of performance should be determined to fit the location, situation, etc.

Pretests over concepts and subject matter of certain chapters, which may be of a review nature may be given to determine if student could proceed.

Specific and clear instructions should be given to the class as to the implementation and conduct of this program. Good class notes, individual projects, supplemental reading should be implemented.

	Give pretest over Chap 1	If pretest score is satisfactory proceed to lesson 7
1	To review mathematical statements	R, 1-1 5/1,3,7,11,13,15 0 4/1-26
2	To review concepts of sets, symbols used, and the real numbers; use of variables, quantifiers and open sent.	R 1-2 and 1-3 1-2, 9-10/1,7,9,11,15,17,21,23,27,29 1-3, 14/3,5,9,13,15,19 R, 1-4 Take Test #1
3	To review conditional and converse statements	1-4, 18/1-17 odd S, 15/29,31 R, 1-5
4	To review and relearn the basic axioms and properties of real numbers	1-5, 25/3,7,9,11,15,17,19,23,25 S, 14/21,23,25,27 R, 1-6
5	To review theorems of addition and their proofs. To be able to prove some simple extensions of some properties.	0, 30-31/1-23 odd, 25-28 all 1-6, 32/7,9,11,15,19,21,23 S, 24/33,35 R, 1-7 FS #210
6	To review basic theorems about multiplication. To be able to set up some proofs of theorems about multiplication	1-7, 37/7,11,15,17,19,21 Take Test # 2
	If average of test # 1 and #2 is below _____	Complete 41/1-57 Then take retest over Chap. 1
7	To learn what constitutes an Abelian Group and be introduced to abstract group theory	R, pp. 44-47 46/1-12 all, 13,15,17
	Take pretest over contents of Chap 2 If score is satisfactory go to lesson 18	
8	To review the required properties of operation for solving simple equations	R 21 0 54/1-30 55/3,7,13,19,21,25,27,29 R 2-2 FS #214
9	To review operations with polynomials	2-2 60-61/1,5,9,13,---,41,45 S, 31/ write 25,27 R 2-3 FS #214
10	To review the operational properties of equality used for transformations to produce equivalent equations	2-3 65-66/1,3,7,11,17,21,27,29,37,41 S 54/write 25 R 2-4(study problem examples carefully)

Lesson No	OBJECTIVE	ACTIVITY
11	To apply algebra to problems by developing a sequence of action. 1. Read problem, what is to be found? 2. Select variable. 3. Write equation based on given information. 4. Solve the equation or open sentence. 5. Check result with the problem.	2-4 73-75/5,7,11,19,23,25,29 S 66-67/31,33 FS #013 and #210.
12	To continue to improve problems solving ability	2-4 73-75/9,13,17,21,27,30,31 67/37,39 R 2-5 <u>Take test # 3</u>
13	To review the axioms of inequality and application to problems	2-5 81-82/15,9,13,17,21 82-83/1,5,7 S 75/26,32
14	To continue to practice and improve problem solving ability with inequalities	S 81-82/3,7,11,15,19,23 82-83/3,9,11 67/45,47 R 2-6
15	To review the absolute value in equalities and use of union and intersection of sets to facilitate finding solution-sets	2-6 87-88/1,5, 9,11,13 S 67/49,51 R 2-7                      FS # 004
16	To review and be able to use the theorems of order	2-7 92/1,3,7,11,13,15 S, 88/25,27 <u>Take test # 4</u>
	If average of test 3 and 4 is below _____	Complete #1-36 pages 95-96 Then take Chap. test #2
17	To learn the symbols used in math logic	Read and study pp. 99-103 101/1-17 odd
18	To learn what a sequence is and the characteristics of the arithmetic sequence	R 3-1      0, 107/1-15 odd 107-108/1,3,5,7,9,11,13,15,17,19 R, 3-2                      FS #005
19	To discover and use the formula for the general term of an A.P and to find the arithmetic mean	3-2, 111-112/1,5,7,9,13,15,19 S, 108/21,23,27 R, 3-3
20	To find the sum of an arithmetic series and use of and interpreting the summation sign (sigma)	3-3, 117-118/1,5,9,11,13,17,21,29 S, 112/21,25,27 112-113/1,5,9,

Lesson No.	OBJECTIVE	ACTIVITY
21	To continue to develop mastery of use of the AP by solving problems involving the Arith. Prog.	S, 117-118/19, 25, 27, 31 118-119/1, 7, 11, 13 113/11 112/29 <u>Take test # 5</u> R 3-4
22	To learn what a geometric sequence is and discover and use the formula for the nth term of a G.P.	0, 122/1-16 123/3, 5, 7, 9, 13, 17 123-124/1, 3, 5, 7 S, 112/31 FS # 006 R, 3-5
23	To be able to find the geometric mean (s)	3-5, 128/3, 5, 7, 9, 13, 17, 19, 21 S, 124/9, 10, 11 R, 3-6 <u>Take test # 6</u>
24	To learn what a geometric series is and to discover and use the sum formula. The summation notation should be understood.	3-6, 0 131/1-10 131/5, 9, 13, 17, 19, 25 S, 128/23, 25 123/15 R, 3-7
25	To understand meaning of limit of a series and be able to determine the boundedness.	3-7, 136-137/1, 5, 9 S, 131/7, 15, 27 132/1, 5, 9 R, 3-8
26	To be able to determine the sum of the infinite geometric series	3-8, 141/1, 5, 9, 13, 21, 25 S, 137/3, 7 <u>Take test # 7</u> FS # 204
If average of tests 5, 6, and 7 is below _____		Complete pages 145-146/1-25 Then take Chap. test # 3
27	To review function and relation, emphasizing domain and range.	R, 4-1 152-153/1, 5, 7, 11, 13, 17, 21, 25, 27 R, 4-2 FS # 212
28	To be able to draw mapping diagrams of functions and to graph functions on the coordinate plane.	4-2, 153 -159/1, 3, 7, 9, 15, 21, 23 S, 152-153/3, 11, 15, 19, 23 R, 4-3
29	To review the graphing of the linear equation $Ax + By = C$	4-3, 163/5, 9, 11, 15, 19, 21, 27, 31 S, 159/25, 27, 29 R, 4-4
30	Review the graph of the linear inequality	4-4, 167/5, 9, 17, 19, 23, 27 S, 163/25, 33 159/31 153/31 R, 4-5 <u>Take Test # 8</u>
31	To review the various aspects of slope of a line and methods of obtaining the slope	4-5, 173-174/1, 5, 11, 13, 15, 19, 25, 29 S, 167/25 153/29, 33 R, 4-6

Lesson No.	OBJECTIVE	ACTIVITY
32	To be able to write the equation of a line, given 2 points on the line, slope and 1 point or slope and the y intercept.	4-6, 178-179/1,5,9,11,17,19,23,25 S, 173/17 174/33 R, 4-7 FS # 208
33	To relate the linear function to direct variation, to find the constant of variation(slope) and to form the proportion of variation.	4-7, 184-185/1-13 odd S, 178-179/7,15,27,31 R, 4-8 Emphasize the different forms of function notation.
34	To relate functions to special forms, using different subsets of the domain such as the <u>step</u> function	4-8, 188-189/1,3,5,9 S, 186/3,7,9 179/35,39 173/17 Read pp.194-195 Take Test # 9
35	To be able to graph systems of linear equations in the plane and produce a solution set.	R, 5-1 202/1,4,9,11,13 R, 5-2
36	To solve systems of linear equations by transformation involving elimination of one variable by substitution, addition or subtr.	5-2, 208-209/1,5,9,13,17,21,25,33 S, 202/15,17 R, 5-3
37	To show ability to solve problems using 2 variables and forming a pair or system of equations.	5-3, 214,216/7,11,13,15,19,23,27,31 S, 209/37 R, 5-4
38	To show the application of systems of inequalities to the very important present day linear programming method.	5-4, 220-221/9,11,17,21,27,31 221/1 S, 215/29 Take test #10 R, 5-5
39	To represent on the plane the 3 dimensional concept by plotting ordered triples with respect to the 3 dimensional axes.	5-5227-228/1,5,9,13,15,19,23 S, 222/5,6
40	To continue practice with the graphing concept	S, 227-228/7,11,17,21 221/29,33 222/7,8 R, 5-6
41	To be able to represent the equation of a plane as a 3 dimensional drawing on a plane using intercepts and traces.	5-6, 234/1,5,9,13,17,21,25 S, 221/3 214/14 R, 5-6

Lesson No.	OBJECTIVE	ACTIVITY
42	To continue practice with sketching the plane in 3 dimensional space.	S, 234/3,7,11,15,19,23,27. 221/35 R, 5-7
43	To be able to represent graphically 3 planes and their pt(s) of intersection. Also be able to make the algebraic transformations to give the solution set corresponding to the point of intersection of the planes.	5-7, 240-241/1,5,9,13,17,21 S 241-242/1,5 S 234/39
44	To continue practice with finding solutions of systems of equations in 3 variables and to apply the methods to some verbal problems.	S, 240-241/7, 11,23,25 242/7,9 221/28  Take Test # 11
	If average of tests, 8,9, 10, and 11 is below _____	Complete pp. 244-245/1-23 and take Chap. test # 5.
45	To review and be able to apply the laws of positive and negative exponents to multiplication and division and powers.	6-1, 255-256/1,5,9,13,17,21,25 29,33,35,39. R 6-2
46	To review and reinforce ability to multiply polynomials with particular attention to the special forms such as: $(a - b)^2$ , $(a + b)(a - b)$ , and $(a + b)^2$ .	6-2, 258-259/1,5,9,13,17,21,25,29,33, 37. S 256/41 R. 6-3
47	To be able to factor given polynomials, particularly the trinomial at sight whenever possible.	6-3, 264/1,5,9,13,21,25,29,33,37 S, 256/43
48	To continue practice factoring polynomials	S 264/3,7,11,27,31,39,43,45 256/45 R 6-4
49	To be able to apply factoring ability to the solving of equations and problems	6-4, 268/1,5,9,13,17,21,25,29, 269/3,5 S, 264-265/41,53,57 FS # 207
50	To continue practice in factoring of polynomials and solving of problems involving the factoring of equations	S 268-269/11, 15,19,23,33 270/ 13,15 255/59,61 R 6-5 Take Test 33



## Lesson

No.	OBJECTIVE	ACTIVITY
51	To apply factoring ability to solution of the inequality.	6-5, 272/1,3,5,7,9,11,13 S 270/17; 263-269/35,37 R 6-6
52	To be able to divide with polynomials as dividend and divisor	6-6, 276/3,7,11,13,15,21,27,31 S 272/17; 269/39 R, 6-7
53	To be able to simplify rational algebraic expressions(fractions)	6-7, 279-280/5,9,13,17,19,25,27,29 S 277/33 272/15,16 272 R 6-8
54	To be able to find products and quotients of rational algebraic expressions.	6-8 282-283/1,3,7,9,15,17,19,23 S 280/33; 276/25 R 6-9 <u>Take test # 14</u>
55	To be able to add and subtract rational algebraic expressions	6-9, 286-287/ 3,5,9,13,17,21,25, 29, 31,37 S 282-283/11,25
56	To continue practice with addition and subtraction of algebraic fractions	S 286-287/11,19,27,35,39,41,43 283/27; 280/28, 35 R 6-10
57	Application of the operations with fractions to equations and problems with rational coefficients	6-10, 290-291/3,7,9,11,15,21 292/ 5,7,11 S 277/35
58	To continue practice and problem solving of rational algebraic equations	S 290-291/17, 23,25,27,33,35 292-293/9,17,19 R 6-11
59	To show application to fractional equations of previous principles. (variables appear in the denominator)	6-11, 296-297/5,13,19,21,27 298-299/5, 9, 11, 15 S; 283/29
60	To continue practice with fractional equations	S 297/11,23,25,29 298-299/7, 13,19,31 287/49,53 <u>Take Test # 15</u>
	If average of tests P, 14 and 15 is below _____	Complete pp301-302/1-23 and take Chap. test #6
61	Be able to graph the power function and its application as direct proportion to many types of important physical science problems.	R 7-1 0,309-310/1-19 odd 7-1, 310/1,3,5,7,11,15,17,21 R 7-2

Lesson  
No.

OBJECTIVE

ACTIVITY

62	To be able to determine the kinds and no. of roots of radicals and estimate roots from a graph of the power function	7-2 314-315/3,7,11, 15,19 S 310/9,13,19 R 7-3
63	To be able to determine the rational roots, if any, of a polynomial function	7-3, 319/5,9,13,15,19 S, 314-315/9, 13, 21 311/3,5 R 7-4
64	Review and be able to change decimal numerals to rational numbers, and to estimate the decimal value of an irrational number.	7-4 324/1,5,9,13,15,21,25,31 S 320/17; 312/9 R 7-5
65	Be able to operate with real numbers as radicals (simplified form)-Multiplication and division	7-5, 328/1,3,7,9,15,19,23,27,33,37 S, 324/19,27,33 R 7-6 <u>Take Test # 16</u>
66	Be able to add and subtract with simplified radical expressions	7-6, 331-332/1,5,9,13,15,17,19,21,25,27. S, 329/49,51; 330/5 R, 7-7
67	Be able to simplify products and quotients which contain radicals	7-7, 333/1,3,7,9,23,27 S, 331-332/15,19,23,29,31,35,37 328/45,47 R, 7-8
68	Be able to solve equations containing radicals	7-8, 336-337/5,7,11,13,17,25,27,29 S 334/29,31,37,41,43 R, 7-9
69	Be able to solve a quadratic equation by completing the square. Use completing the square to derive the quadratic formula	7-9, 340-341/3,7,11,13,17,19,25,29,31 S 337/35,37,41
70	To gain skill in application of the methods of solving a quadratic equation.	S, 340-341/5,15,21,27,35,41,45,47 342/3,7; 337/31 R 7-10
71	To learn to use the relation between roots and coefficients of a quadratic equation.	7-10, 345/3,9,11,17,19,21,23 S 334/38,47 <u>Take test # 17</u>
	If average of tests 16 and 17 is below _____	Complete pp 347-348/1-15 and take Chap. test # 7



Lesson  
No.

## OBJECTIVE

## ACTIVITY

72	To be able to graph and determine characteristics of the Quadratic function	R, 8-1, 0 356/1-17 odd 357/1,5,9,13,15,19,23 R, 8-2
73	Further work with the function $y = a(x-h)^2 + k$ .	8-2, 360-361/1,5,9,13,17; 361/1 S 357/7,11,17,21,25; FS#014 R 8-3
74	To be able to solve the quadratic inequality by factoring and graph	8-3, 364/1,5,9,13; S, 361/3,7; 360-361/3,7,15; 357/20 R, 8-4; <u>Take test # 18</u>
75	To know and understand the imaginary unit (i) and to simplify radical expressions with neg. radicand.	8-4, 368-369/1,5,9,13,17,21,25,29,33, 37,39. S, 364/3,15 R, 8-5
76	To identify the complex no. $(a+bi)$ and how to work with them in add. and sub.	8-5, 373/1,5,9,13,17,21,25,29,33 S 368-369/3,7,11,15,27,41,43 R, 8-6
77	To be able to determine the nature of the roots of a quadratic equation by use of the discriminant.	8-6, 377/1,5,11,15,23 S, 373/7,15,19,27,31 R, 8-7 <u>Take test # 19</u>
78	To be able to determine values of a polynomial function by synthetic sub.	8-7, 380-381/1,5,9,17,21,25. S, 377/7, 13,17,25; 364/7 R, 8-8
79	To understand the remainder theorem and its corollary, the factor theorem and be able to use the factor theorem to find the zeros of a function.	8-8, 384/1,5,9,13,17,21,25,29,31 S, 380-381/15,23 364/11 R, 8-9
80	Be able to apply the fundamental theorem of algebra and to estimate real roots of a polynomial when they are not rational.	8-9, 388-389/1,5,7,17, 21. S 384-385/3,7,11,15,19,23
81	Continue to practice in finding estimations of roots.	S, 388-389/3,11,19,23 <u>Take test # 20</u>
	If average of tests 18,19 and 20 is below _____	Complete pp. 393/1-13. and then take Chap. test # 8
82	To understand the extension of the laws of exponent to rational numbers and to write radicals in exponential form	R 9-1: 400-401/ 1,5,7,11,13,17,21, 25,29,33,41. R, 9-2; FS# 215

Lesson No.	OBJECTIVE	ACTIVITY
83	To understand that all real numbers may be used as exponents and to be able to graph the exponential function.	9-2, 404/1,5,9,15,17,21 S, 400-401/3,9,15,23,31,35 R, 9-3
84	To identify the inverse of a function and be able to graph a function and its inverse.	9-3, 407/1,5,9,13 S 404/3,11,19,23 401/19,27,37 R 9-4
85	To identify specifically the inverse of the exponential function as the logarithmic function and be able to change from one form to the other.	9-4, 410/1,5,9,13,17,25,29; S, 407/3,7; 404/25; R, 9-5 <u>Take Test # 21</u>
86	To establish the meaning and use of the terms, precision accuracy of a measurement and to review scientific notation.	9-5, 413-414/1,5,9,13,15,17,23,27,31; S, 410/11,15,27,35 R 9-6
87	To understand common logarithms, the characteristic, mantissa, and the anti-logarithm and use table.	9-6, 417/1-25 odd S 413-414/3,17,19,21,25,29,33 R, 9-7
88	To be able to use interpolation to find mantissas of the logs of a significant	9-7, 419/1,3,5,13,15,17 S, 414/28; 410/7,9,13; R 9-8; <u>Take Test # 22</u>
89	To learn how to find the product and quotient of decimal numbers using logarithms.	9-8, 423/5,11,17,23,25,33,35 S, 410/3,23,33 R, 9-9
90	To learn how to find powers and roots of decimal numbers by using logarithms.	9-9, 426-427/1,3,7,23,25,29,35. 427/1 S, 410/39 R, 9-10

Lesson no.	OBJECTIVE	ACTIVITY
91	To be able to solve equations requiring the use of logarithms.	9-10/1,5,9,13,17,21 S 428/7; 426/27; 401/45 FS#011
92	Additional practice in solution of various types of equations using logarithms.	S 430/7,15,19,23; 488/5; 427/35,37,41 423/29 TEST #23
	If average of tests #21,22, and 23 is below _____	Complete page 432, #1-10 Take chapter test #9
93	Review of coordinates in the plane, and developing the distance formula	R 10-1 437-438/1,5,9,13,17,19,23,29 R 10-2
94	To review slope and the point slope form and establish and use the slope relationship of perpendicular lines.	10-2 441/1,5,9,13,17, S 437/3,15,21,25 R 10-3
95	To find the equation of the circle using the distance formula and definition of the circle as a locus or a set of points. Be able to find the center and radius of a circle.	10-3 443-444/1,5,9,13,17,21,25 S 441/7,11 437/7 R 10-4 FS#015
96	Be able to find the equation of the parabola using the foci, directrix and distance. Recognize and describe the parabola whose equation is given.	10-4 448-449/1,5,9,17,21 S 443/3 441/15 437/7 R 10-5 Take Test#24
97	Be able to find the equation of the ellipse using focal points and distance. Identify and describe the ellipse from the given equation.	10-5 452-453/1,5,9,13,15,17 448-449/7,15,19 443/16 10-6
98	To determine the equation of the hyperbola from the given information and to identify and describe the equation of the hyperbola.	10-6 457-458/1,5,11,13,15 452-453/3,7,17 448-449/3,13 R10-7
99	Understand the inverse variation and application of the hyperbolic function and be able to use the inverse variation.	10-7 461-462/1,3,5,9,13,17 FS#203 S 457-458/3,14 453/16,19 R 10-8 Take Test#25

Lesson No.	OBJECTIVE	ACTIVITY
100	Be able to graph the different quadratic equations and determine the solutions.	10-8 464-465/1,5,9,13,17 S 462-463/1,5 462/11,15; 458/19
101	Continue the graphin of quadratic systems to indicate the number and the approximate solutions.	S 464-465/5,15,23 463,3,7; 462/19,21 453/23 4444/29 R 10-9
102	To learn how to algebraically determine the solution sets of linear quadratic systems by substitution.	10-9 466-467/5,7,9,13,21 467/1,3 464-465/11,21 R 10-10
103	To learn how to solve quadratic-quadratic systems by combining the equations in some manner and eliminating any one variable.	10-10 469-470/1,5,9,13,17 S 467-468/5,7 466-467/15 462/23
104	Review the methods of solution of systems and additional practice in problem solving.	S 469-470/3,7,15 470-471/1,5,7 467/19; 461/7 Test #26
	If average of tests #24,25, and 26 is below _____	Complete page 473 # 1-13: and Chapter Test #10
105	To review the geometric angle and correlate it with rotation and the angle in standard position.	R11-1; 484-485/1,3,7,9,11,13,17,21 R11-2
106	To review the measurement of angles in degrees and to learn the radian measure of an angle and the relationship of the radian to degree measure.	11-2 487/1,5,9,13,17,21,25,33 S 484/5,15,19 R 11-3
107	To learn the coordinate definition of the sine and cosine function and why they are called circular functions.	11-3 493-494/1,3,5,9,13,17,21 S 489/7,23,31 485/23 R 11-4 Test #28

Lesson No.	OBJECTIVE	ACTIVITY
108	To learn the values of the trigonometric functions for special angles: 30 ,45 ,60 , 90 , etc., including multiples of 360 .	11-4 498/1,3,7,9,13,15,17 S 493-494/11,15,23 489/19 11-5
109	To learn to use trigonometric tables and to find the approximate values of trigonometric functions.	11-5 502/1,5,9,13,17,21,25,33,41,45 S 498/5,11 R 11-6
110	To learn how to find reference angles and arcs for angles greater than 90 or radians.	11-6 505-506/1,5,9,11,13,15,19,25,33,39,45, S 498/19 R 11-7 <u>Test #29</u>
111	To be able to correctly graph the the sine and cosine functions, learning the meaning of amplitude and period.	11-7 512/1,5,6,8,13,15 S 505-506/3,27,45,49 R 11-8 FS #202
112	To learn by definition, in terms of sine and cosine, the tangent, cotangent, secant and cosecant functions.	11-8 516-517/1,5,9,11,15,17,19,31,37 S 512/3,7 R 11-9 FS #224
113	To learn to use all six functions, as required, to solve right triangles and their application to problems requiring solution of right triangles.	11-9 521/3,7,9,11,13,15 521-522/1,5 S 516-517/7,21,33 506/53
114	Continue practice in solving right triangles utilizing angles of elevation and depression.	S 522-523/3,9,11 521/5,17,19 517/13,23,39,43 Test #30
	If average of tests #28,29, and 30 is below _____	Complete Page 525. #1-15 and Chapter Test 11
115	To learn the eight fundamental identities and how to use them in simplifications of trigonometric expressions.	R12-1 529-530/1,3,5,7,9,13,17,19 R 12-2

## Lesson

No.	OBJECTIVE	ACTIVITY
116	To learn to use the identities in proofs and development of other identities plus simplification of some complicated trigonometric expressions.	12-2, 532-533/1, 5, 9, 13, 17, 21, 23 S 529-530/11, 15, 23 R 12-3
117	To learn why special formulas are needed for sums and differences of trigonometric functions of an angle and what the cosine of a sum or difference is.	12-3 540-541/1, 5, 9, 13, 17, 21, 25 S 532-533/7, 15, 25 R 12-4 <u>Take test #31</u>
118	To use the previous formula to find the formulas for the sine and tangent of a sum or difference and to use them	12-4 547-548/1, 5, 9, 13, 17, 25, 29 S 541/11, 2k, 29 533/19 R 12-5
119	To derive more identities for double and half-angle formulas.	12-5 554-555/1, 5, 9, 13, 17, 23, 31, 39 S 547-548/11, 33, 39, 41, 53 R 12-6 <u>Take test #32</u>
120	To develop the Law of Cosines and to apply the Law to the solution of oblique triangles.	12-6 558/1, 3, 5, 559/1, 3 S 554-555/11, 19, 25, 35, 41 FS#222
121	To develop and use the Law of Sines, and to apply the Law to oblique triangles.	12-7 563-564/1, 3, 5, 11, 19 564-565/1, 5 S 554/29, 43 548/47, 49 FS #223
122	To continue practice with applications of the sine and cosine laws to examples and problems.	S 563-564/7, 15, 21 565/7; 558/9 555/47, 63, 65 548/35, 45 <u>Take test #33</u>
	If average of tests 331, 32, 33 is below _____	Complete page 566 #1-10 and take Chap. <u>test #12</u>
123	To learn to graph and use notation for the inverses of the periodic functions and principal value	R 13-1 574-575/1, 3, 7, 9, 13, 15, 17, 25, 29, 33 R 13-2

Lesson No.	OBJECTIVE	ACTIVITY
124	To be able to find solutions to equations involving circular and Trigonometric functions.	13-2 579/1,5,9,13,17,23,27 S 574-575/5,11,21,27 R 13-3 Test #34
125	To learn what polar coordinates are plus using them to produce polar graphs, and the interchange with Cartesian Coordinates.	13-3 583-584/1,5,9,13,17,21,25,29,31,33,35 S 579/21,33 R 13-4
126	To apply the polar form to complex numbers and become acquainted with and to use De Moivre's Theorem.	13-4 590-591/1,5,9,11,17,21,25,31 S 583-584/7,15,19
127	Continue practice with complex numbers and use of De Moivre's Theorem.	S 590-591/7,13,19,23,33 584/27,37,43 579/3,11
128	To learn how vectors may be used to represent directed line segments and notation.	R 13-5 595/1,3,5,9,13,15 FS #213 S 590/3,15,27 579/15,25 Test #35
	If average of tests #34 and 35 is below _____	Complete Chapter Test #13
129	To learn the Fundamental Principle of counting, as related to Cartesian Products.	14-1 601/1,3,7,9,11,13,15 R 14-2
130	To learn the principle of permutations and be able to find linear and circular permutations.	14-2 604-605/1,5,9,13,17,21,25 S 601/5,17 R-3
131	To continue practice in finding the number of distinguishable permutations	14-3 606-607/1,5,9,13,17 S 604-605/3,7,11,27, 601/8 R 14-4 Test #36
132	To learn to count subsets, to determine the number of combinations that can be formed from a given set.	14-4 609-610/1,3,7,11,13,15 S 606-607/3,15 605/15,19 R 14-5

Lesson No.	OBJECTIVE	ACTIVITY
133	To be able to find the number of combinations that can be formed from several sets.	14-5 611-612/1,3,5,7,11 S 609-610/5,17 606-607/7,15 R 14-6
134	To develop and apply the binomial theorem, to expand a binomial and also to find a given term of a binomial expansion.	14-6 614/1,5,9,13,15 S 612/9,12 609/9,10
135	To continue practice with the binomial theorem and its application.	S 614/3,11,17,19 612/13; 609/6,12 606/11 605/23 R 14-7
136	To note the application of Pascal's Triangle and combinations to the finding of coefficients of terms in the binomial expansion.	14-7 616/1,5,7,9 S 611/2,8 609/2,8 Test #37
137	To learn to properly list a sample space of events and select specified events.	14-8 618-619/1,3,7,9 S 616/3,6,8 601/10,14
138	To continue practice with events and sample spaces.	S 618/2,5,8,10 614/7,14,16 607/14,16 R 14-9
139	To understand the meaning of mathematical probability and develop, and use, the basic formula for probability.	14-9 621/1,5,9,13 S 618/6 616/10 610/14 605/23 R 14-10
140	To determine what is meant by mutually exclusive events, the formula and application of the formulas.	14-10 624/1,5,7, S 621/37 614/12 618/4 610/16
141	Continue application of finding probabilities.	S 624/3,6,8 614/18 622/11 611/4,6 610/18 R 14-11



Lesson No.	OBJECTIVE	ACTIVITY
142	To determine the difference between independent and dependent events and how to determine probabilities of said events.	14-11 626-627/1,5,9,11 624/2,4 621/4,6 610/19
143	To continue practice with probability problems.	S 626-627/2,3,7,10 622/10,15 612/15 Test #38
144	If average of tests #36,37,38 is below _____	Complete Page 629 #1-11 and Chapter Test #14
145	To understand matrix, entry, dimension. How to find sum of matrices, rules for adding the identity and the inverse	R 15-1 638/1,5,9,13,17,21,25 R 15-2
146	Be able to find product of a scalar and a matrix	15-2 641/1,5,9,13,17,21 S 638/3,11,23 R 15-3
147	Be able to find the product of two matrices.	15-3 645/1,5,9,13,17,21 S 641/3,15; 638/27 R 15-4
148	To investigate and discover properties of matrix multiplication.	15-4 648/1,5,9,13,17,21,25 S 645/7,15,19
149	To apply and review operations with matrices.	S 648/3,7,15,19,23 645/11,15; 641/7,19 R 15-5
150	From the give definition of the determinant, find the determinant of given 2x2 and 3x3 matrices, by expansion by minors.	15-5 652-653/1,5,9,13,17,21,25 S 648/11 645/3 R 15-6
151	Be able to find the inverse of a matrix using the determinant function	15-6 656-657/1,5,9,13,17,21,25 652-653/11,15,23,27 R 15-7

Lesson  
No.

OBJECTIVE

ACTIVITY

152	Be able to solve linear systems by matrix solution and by Cramer's Rule.	15-7 661/1,5,9,17,21 S 656-657/7,11,15,23 653/29 Test #40
153	Continued practice with matrices. To review and strengthen ability to work with matrices and their determinants.	S 661/7,15,19 656-657/3,19 652-653/7,31 648/27 645/23
	If average of tests #39,40 is below _____	Complete Page 663 # 1-10 and Test # 15

PARAMETRIC FORM OF EQUATIONS  
IN COORDINATE PLANES

VI. Space Geometry

G12-S

1. The student can write out and graph parametric equations, in keeping with his ability.

Readiness: The student is familiar with algebraic equations and graphing in a coordinate system.

- 1a. Am. Bk. Co.; Kline, et. al., FOUNDATIONS OF ADVANCED MATHEMATICS, pp 271 - 274 c 1959

- b. AW.; Shanks, et. al., pp 15 - 33, 79 - 90; c 1965, PRE - CALCULUS MATHEMATICS

VECTOR FORM OF LINES AND PLANES IN SPACE

VI. Space Geometry

G12-S

2. The learner can identify, by defining and graphing, the vector form of lines and planes, within the limit of his ability.

Readiness: The student is familiar with scalar quantities and plotting points on a coordinate system.

- 2a. A.; Shanks, et. al., pp 40 - 57 PRE- CALCULUS MATHEMATICS; c 1965

DIHEDRAL AND POLYHEDRAL ANGLES

VI. Space Geometry

G12-S

3. Given a set of half planes, the student can construct dihedral and polyhedral angles, in keeping with his ability.

Readiness: The student recognizes planes, half-planes and lines.

- 3a. AW; PRE-CALCULUS MATHEMATICS; Shanks, et. al., c 1965; pp 72-74
- b. Am. Bk. Co.; Kline, et. al. c 1959 FOUNDATIONS OF ADVANCED MATHEMATICS; pp 18-25

FREQUENCY DISTRIBUTION, CUMULATIVE  
FREQUENCY AND PERCENTILES

VI. Space Geometry

G12-S

4. Given a list of data, the learner can compute the frequency distribution, cumulative frequency and classify the percentiles, within the limits of his capabilities.

Readiness: The student is familiar with arithmetic averages.

- 4a. Am. Bk. Co.; Kline, et. al. c 1959; FOUNDATIONS OF ADVANCED MATHEMATICS; pp 403-408

VARIANCE AND STANDARD DEVIATION

IX. Statistics

G12-S

5. Given a distribution, the student can compute the variation and standard deviation, in keeping with his ability.

Readiness: The student is familiar with frequency distribution, mean, mode, and median.

- 5a. Am. Bk. Co.; Kline, et. al. c 1959; FOUNDATIONS OF ADVANCED MATHEMATICS; pp 410-413

PERMUTATIONS AND COMBINATIONS

IX. Statistics

G12-S

6. Given a set of objects, the student can arrange these objects according to  $P(n,r)$  and  $C(n,r)$ , commensurate with his ability.

Readiness: The student is familiar with the fundamental principles of counting, and ordered pairs.

- 6a. AW.; Shanks, et. al. c. 1965; pp470-75 PRE-CALCULUS MATHEMATICS;  
b. Am. Bk. Co.; Kline, et. al. c 1959; FOUNDATIONS OF ADVANCED MATHEMATICS; pp 370-378

PROBABILITY - SETS

IX. Statistics

G12-S

7. Given two events, the student can apply the addition and multiplication theorems to predict the probability of the events occurring, within the limit of his ability.

Readiness: The student is aware that an event can occur or not occur and if the event occurs, it can happen in more than one way.

- 7a. Am. Bk. Co., Kline, et. al., c 1959;  
FOUNDATIONS OF ADVANCED MATHEMATICS;  
pp381-387

THE WRAPPING FUNCTION

X. Trigonometry

G12-S

8. The student can correctly define the wrapping function when requested to do so, according to his ability.

Readiness: The student is familiar with number line and one-to-one, one-to-two, etc., correspondence.

- 8a. A.I.; Shanks, et. al. c 1965,  
PRE-CALCULUS MATHEMATICS; pp 187-191

## CIRCULAR FUNCTIONS

X. Trigonometry

G12-S

9. The student can define, in writing, the three basic circular functions and their reciprocals, in keeping with his ability.

Readiness: The student is familiar with functions.

- 9a. AW.; Shanks, et. al. c. 1965;  
PRE-CALCULUS MATHEMATICS; pp 191-200
- b. Am. Bk. Co., Kline, et. al., c 1959  
FOUNDATIONS OF ADVANCED MATHEMATICS;  
pp 65-78

## GRAPHS OF TRIGONOMETRIC FUNCTIONS

X. Trigonometry

G12-S

10. The student can demonstrate his ability to graph the basic trigonometric functions, including variations in amplitude and phase shift, within the limit of his ability.

Readiness: The student can graph algebraic functions; the student is familiar with concepts of basic trigonometric functions.

- 10a. AW., Shanks, et. al., c 1965;  
PRE-CALCULUS MATHEMATICS; pp 200-1
- b. Am. Bk. Co., Kline, et. al., c 1959  
FOUNDATIONS OF ADVANCED MATHEMATICS;  
pp 106-119

TRIGONOMETRIC IDENTITIES

X. Trigonometry

G12-S

11. Given a trigonometric expression, the student can simplify the expression or sentence by applying the fundamental identities, within the realm of his ability.

Readiness: The student knows the fundamental trigonometric identities.

11a. Am. Bk. Co. Kline, et. al., c. 1959; FOUNDATIONS OF ADVANCED MATHEMATICS; pp 120-129.

b. AW., Shanks, et. al., c 1965; PRE-CALCULUS MATHEMATICS; pp 248-56.

TRIGONOMETRIC EQUATIONS

X. Trigonometry

G12-S

12. Given a list of trigonometric equations, the student can solve the equations, within the limit of his ability.

Readiness: The student is familiar with the solution of algebraic equations and a knowledge of trigonometric functions.

12a. AM., Kline, et. al. c., 1959; FOUNDATIONS OF ADVANCED MATHEMATICS; pp130-135

b. AW., Shanks, et. al. c., 1965; PRE-CALCULUS MATHEMATICS; pp 283-5



FUNCTIONS OF THE SUM , DIFFERENCE,  
DOUBLE AND HALF ANGLE FORMULAS

X. Trigonometry

G12-S

13. Given an angle or angles, the student can calculate their sum, difference, double and half angles by using the appropriate formula, in keeping with his ability.

Readiness: The student is acquainted with the circular trigonometric functions.

- 13a. AW., Shanks, et. al. c., 1965;  
PRE-CALCULUS MATHEMATICS;  
pp 252-263
- b. Am. Bk. Co.; Kline, et. al.; c 1959  
FOUNDATIONS OF ADVANCED MATHEMATICS;  
pp 136-152

INVERSES OF TRIGONOMETRIC  
FUNCTIONS AND RELATIONS

X. Trigonometry

G12-S

14. The student can identify the inverse circular functions in writing and given a value, can compute the corresponding number of radians or degrees, within the limit of his capacity.

Readiness: The student is acquainted with the circular functions and algebraic inverses.

- 14a. AW., Shanks, et. al., c 1965;  
PRE-CALCULUS MATHEMATICS;  
pp 276-283
- b. Am. Bk. Co.; Kline, et. al. c 1959;  
FOUNDATIONS OF ADVANCED MATHEMATICS;  
pp 153-160

POLAR COORDINATES

X. Trigonometry

G12-S

15. Given a set of ordered pairs of the form  $(r, \theta)$ , the student can plot the ordered pair on a polar coordinate system, within the limit of his ability. .

Readiness: The student is familiar with radii and angles, ( angle measurement).

- 15a. AW., Shanks, et.al. c. 1965;  
PRE - CALCULUS MATHEMATICS;  
pp 370-382

- b. Am. Bk. Co.; Kline, et. al. c. 1959;  
FOUNDATIONS OF ADVANCED MATHEMATICS,  
pp 274-285

SINE AND COSINE LAWS

X. Trigonometry

G12-S

16. Given certain parts of a triangle, the learner can solve the triangle by applying the sine or cosine law, within the limit of his capability,

Readiness: The learner is familiar with the trigonometric functions.

- 16a. A1., PRE-CALCULUS MATHEMATICS;  
Shanks, et. al. c. 1965; pp 236-40

- b. Am. Bk. Co. Kline, et. al. c 1959;  
FOUNDATIONS OF ADVANCED MATHEMATICS;  
pp 179-190

SOLVING RIGHT AND OBLIQUE TRIANGLES  
VIA LOGARITHMS

X. Trigonometry

G12-S

- 17a. Given a set of circular logarithmic functions and anti-logarithms, the student can match the anti-logarithms with the circular functions and vice versa, within the limit of his ability. .

Readiness: The student is familiar with circular functions and logarithms.

- 17b. Given certain parts of a triangle, the student can solve the triangle through the use of logarithms.

Readiness: The learner can convert from circular functions to logarithms and back again.

- 17a. AW.; Shanks, et. al. c 1965;  
PRE-CALCULUS MATHEMATICS;  
pp 233-236

ROTATION AND TRANSLATION TRANSFORMATIONS

XI. Analytical Geometry

G12-S

18. The learner can rotate and translate the conic sections on a coordinate plane, according to his ability.

Readiness: The student is familiar with the conic sections and the coordinate axis system.

- 18a. A.I., Shanks, et. al. c 1965;  
PRE-CALCULUS MATHEMATICS;  
pp 364-369

- b. Am. Bk. Co. Kline, et. al. c 1959;  
FOUNDATIONS OF ADVANCED MATHEMATICS;  
pp 310-314

POLYNOMIAL FUNCTIONS

XIII. Functions

G12-S

- 19a. The student can apply the division, remainder, factor and fundamental axioms of algebra to a given set of polynomials, commensurate with his ability.

Readiness: The student can recognize a polynomial function.

- 19b. The student can graph polynomial functions by sketching, according to his ability.

Readiness: The student is familiar with graphing and polynomials.

- 19a. AW.; Shanks, et. al. c 1965;  
PRE-CALCULUS MATHEMATICS;  
pp 140-145

- b. Am Bk. Co.; Kline, et. al. c 1959;  
FOUNDATIONS OF ADVANCED MATHEMATICS;  
pp 316-329

- 19b<sub>1</sub>. AW., Shanks, et. al. c 1965;  
PRE-CALCULUS MATHEMATICS;  
pp 145-150

- 19b<sub>2</sub>. Am. Bk. Co. Kline, et. al. c 1959;  
FOUNDATIONS OF ADVANCED MATHEMATICS;  
pp 316-329

POLYNOMIAL FUNCTIONS

XIII. Functions

G12-S

20. Given a set of functions, the student can compare and contrast continuity, rational functions, and algebraic functions, according to his ability.

Readiness: The learner is familiar with polynomial functions.

- 20a. AW., Shanks, et. al. c 1965;  
PRE-CALCULUS MATHEMATICS;  
pp 150-163

- b. Am. Bk. Co., Kline, et. al. c 1959;  
FOUNDATIONS OF ADVANCED MATHEMATICS;  
pp 318-319

## EXPONENTIAL FUNCTIONS

### XIII. Functions

G12-S

21. The student can define and graph exponential functions, within the means of his ability.

Readiness: The student is familiar with exponents, functions and graphing.

- 21a. AW., Shanks, et. al. c 1965;  
PRE-CALCULUS MATHEMATICS;  
pp 163-171

- b. Am. Bk. Co., Kline, et. al. c 1959;  
FOUNDATIONS OF ADVANCED MATHEMATICS;  
pp 265-270

## COMPLEX NUMBERS

### XIII. Radicals and Variables

G12-S

22. Given a set of complex numbers, the student can perform the basic operations on them, in both the rectangular and polar coordinate systems, as instructed and within the limit of his capability.

Readiness: The student is familiar with the real and imaginary number systems.

- 22a. Am. Bk. Co., Kline, et. al. c 1959;  
FOUNDATIONS OF ADVANCED MATHEMATICS;  
pp 476-483

- b. AW., Shanks, et. al. c 1965;  
PRE-CALCULUS MATHEMATICS;  
pp 263-272

MATRICES AND DETERMINANTS

XIV. Matrices and  
Determinants

G12-S

- 23a. The student can define matrices and determinants, within the limit of his ability.

Readiness: The student is familiar with systems of linear equations.

- 23b. The student can evaluate second and third order determinants, according to his ability.

Readiness: The student is familiar with rectangular arrays of numbers.

- 23a. AW., Shanks, et. al. c 1965;  
PRE-CALCULUS MATHEMATICS;  
pp 449-450

- b. Am. Bk. Co., Kline, et. al. c 1965;  
FOUNDATIONS OF ADVANCED MATHEMATICS;  
pp 461-462

- 23a. AW., Shanks, et. al. c 1965;  
PRE-CALCULUS MATHEMATICS;  
pp 451-457

- b. Am. Bk. Co., Kline, et. al. c 1959;  
FOUNDATIONS OF ADVANCED MATHEMATICS;  
pp 462-472

CRAMER'S RULE

XIV. Matrices and  
Determinants

G12-S

24. within the limit of his ability, the student can apply Cramer's Rule when solving systems of linear equations.

Readiness: The student is familiar with determinants, and systems of linear equations.

- 24a. AW., Shanks, et. al. c 1965;  
PRE-CALCULUS MATHEMATICS;  
pp 456-461

- b. Am. Bk. Co. Kline, et. al. c 1959;  
FOUNDATIONS OF ADVANCED MATHEMATICS;  
pp 472-473

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University of Illinois Arithmetic Maneuvers on a Lattice, by David Paige  
Project (at Edu.Dev. Center)

University of South Dakota

Film Rental -  
Arithmetic in the Food Store  
Arithmetic in Estimating and Checking Answers

J. Weston Welch  
Box 1075  
Portland, Maine

Games for Learning Mathematics, by Donovan A.  
Johnson \$1.00

Suggest writing for catalog from these people;  
they put out an extensive line of math aids.

Walker Company  
720 Fifth Ave.  
New York, N. Y. 10019

Madison Project Coordinator

Geo-Cards by Donald Cohen

Webster Division  
McGraw-Hill

Problem Cards (\$10 ?)  
A-Block Cards 1-39 )  
Color Cubes 1-17 ) packed together  
People Pieces 1-16 )  
Creature Cards - must be ordered separately  
1-15  
Individual pieces can be replaced in these  
sets, which are most useful for dramatizing  
many mathematics principles.

John Wiley and Sons, Inc.  
New York, N. Y.

Nuffield Mathematics Project (3 sets) (1967)  
I Do and I Understand, Paper, \$1.95

Pictorial Representation (Graphing) \$2.25